

# OPTIMIZATION OF A FUZZY CONTROLLER USING NEURAL NETWORK

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## Abstract

This paper presents a strategy for optimization of a fuzzy logic controller based on a control scheme, which consists of a fuzzy logic (FL) controller and a conventional derivative (D) controller. For this purpose, we first choose a set of membership functions regarding change-in-error  $\dot{e}$ , which represent the feedback of velocity. Then we optimize them using neural network in self-organizing process. To demonstrate the effectiveness of the proposed method, we report a number of simulation results involving both step and tracking control of a nonlinear plant.

## 1 Introduction

It is known that fuzzy controllers are suitable for systems with nonlinear dynamic equations or with unknown structures since empirical knowledge or operator experience can be integrated into the controllers by fuzzy sets and fuzzy rules. The problem with fuzzy logic controllers is that they are difficult to fine tune or optimize.

One of the widely used methods for optimizing a fuzzy logic controller is to define membership functions of linguistic variables and to formulate fuzzy rules by control engineers. These membership functions and rules are stored in computer memory and can not be modified during operations [1][4][6]. Unfortunately, there is no generally applicable principle on determining the membership functions and rules for each particular system with specific requirements. Another approach is to modify the rule base by self-organizing algorithms automatically according to previous responses until the desired control performance is achieved [2][3][5]. In self-organizing control (SOC), modifying the rule base is to optimize a decision table based on a defined performance index. In such a decision table, however, there are a number of elements that should be optimized during operations (a high dimension optimization problem), it is generally a hard problem in optimizing the rule base within small learning times.

In [8]-[9], a FL control scheme for a mechanical manipulator has been proposed, which consists of a FL controller and a conventional derivative (D) controller, as shown in Fig.1. By investigating membership functions regarding change-in-error  $\dot{e}$  in this scheme, the desired control performances have been achieved in presence of joint friction and load changes. In [10], we use the Nelder and Mead's simplex algorithm to optimize these membership functions in self-organizing process. This paper presents a method for optimizing a fuzzy controller based on this scheme. For this purpose, we choose a set of membership functions whose shapes can be adjusted by a parameter vector  $\mathbf{k}$ . Then, we optimize membership functions using neural network in self-organizing process. The proposed approach is suitable for systems that are weakly defined by analytical models and may be more effective than traditional self-organizing control, since it reduces optimizing a decision table (a number of elements in the table) into optimizing membership functions which are represented by the parameter vector. Besides, adjusting membership functions by two or three parameters also can be easily done by control engineers. To demonstrate the effectiveness and robustness of the proposed approach, we report several simulation results involving both step and tracking control of a nonlinear plant.

## 2 Choice of Membership Functions

The input signals to the FL controller in Fig.1 consist of the error  $e = y_d - y(t)$  between the reference and actual values and its change  $\dot{e}$ . The output signal from the FL controller  $u_f$  is determined by the Min-Max inference algorithm and the fuzzy rule base shown in Table 1. The fuzzify section in the FL controller changes the true physical quantities to fuzzy quantities based on membership functions. The defuzzification is carried out by the centroid method. Using this control scheme, the control signal  $u$  for a plant is computed as follows:

$$u = u_f + \int_0^{t_0} u_f dt + K_d \dot{e} \quad (1)$$

where  $K_d$  is the parameter of the D controller. In general, both membership functions of triangular type and exponential type are widely used in FL control. Obviously, changing these membership functions will affect control performance. A membership function of a symmetrical triangle is characterized by its peaking and zero points (belief values of one and zero). Since such membership functions can be adjusted only by shifting their peaking and zero points, they are not flexible enough in modifying their shapes. A membership function of an exponential type  $\mu(\alpha) = \exp[-(\alpha - \alpha_0)^2 / \sigma^2]$  is characterized by its mean  $\alpha_0$  and deviation  $\sigma$ . By increasing  $\sigma$ , the membership functions of this type become flatter. If  $\sigma$  is very large, control performance becomes poor because the corresponding linguistic variables become too "fuzzy". In [6], we have modified the membership functions regarding  $\dot{e}$ , which represent the feedback of velocity, by adding some interpolation points to the triangular type. This idea leads to reduce the number of membership functions to be investigated. The studies in [6] have shown that the shape of membership functions for desired control performance does not fall on the triangular type nor the exponential type. Therefore, the membership functions regarding  $\dot{e}$  in this SOC strategy are defined by cubic polynomials shown in Fig.2b, whose shapes can be modified by shifting the "moving" points along the dashed lines by their parameters shown in Fig.3. The parameters  $k_b$ ,  $k_s$  and  $k_z$ , denoted by a vector  $\mathbf{k} = (k_z, k_s, k_b)$ , are used to adjust the membership functions  $\mu(\text{NB})$  and  $\mu(\text{PB})$ ,  $\mu(\text{NS})$  and  $\mu(\text{PS})$ , and  $\mu(\text{ZO})$ , as shown in Fig.3a-c, respectively. The parameter vector  $\mathbf{k}$  can be changed in the range  $[0.15, 0.85] \times [0.15, 0.85] \times [0.15, 0.85]$ . On the contrary, the membership functions regarding  $e$  and  $u_f$  are chosen to be the triangular type and remain unchanged during operations, as shown in Fig.2a and 2c, respectively.

### 3 Neural Network Structure

In the method, the optimization of the fuzzy logic controller is done by using neural network to search the optimum point of the vector  $\mathbf{k}^* = (k_z^*, k_s^*, k_b^*)$  according to previous responses. Therefore, it is important to define a control performance index. Here, we use the integral-square-error criterion

$$\mathbf{J} = \int_0^{t_0} e^2 dt \quad (2)$$

as a performance index. Since the parameter vector  $\mathbf{k} = (k_z, k_s, k_b)$  has effects on control performance of a manipulator,  $\mathbf{J} = f(k_z, k_s, k_b)$  or  $\mathbf{J} = f(\mathbf{k})$  is used to describe the responses.

Fig. 1 illustrates the fuzzy logic control scheme. In this scheme, a three-layer BP neural network is used to optimize the membership functions for desired control performance in self-organizing process. The input to the neural network is a given samplef ( $\mathbf{k}^*$ ).

The output  $q_j^{[s]}$  of the j-th neuron on the s-th hidden layer is calculated as

$$q_j^{[s]} = g(\sum (w_{ji}^{[s]} * q_i^{[s-1]})) \quad (3)$$

where  $w_{ji}^{[s]}$  is the weight on connection joining i-th neuron in layer (s-1) to j-th neuron in layer s and  $g(\cdot)$  is a sigmoid logistic function

$$g(x) = \frac{1}{1 + e^{-x}} \quad (4)$$

We use the error  $E = f(\mathbf{k}^*) - f(\mathbf{k})$  to modify the weight  $w_{ji}^{[s]}$  by Widrow-Hoff  $\delta$  learning rule. If  $E \leq \varepsilon$ , the output of the neural network is the vector  $\mathbf{k}^* = (k_z^*, k_s^*, k_b^*)$  that determines the optimal membership functions.

### 4 Simulations

In order to demonstrate the effectiveness and robustness of the proposed strategy, by numerical simulations we test time responses for the step, ramp tracking and sinusoidal tracking control of the nonlinear plant

$$\ddot{y} + 2.0\xi\omega\dot{y} + \omega^2 y^2 = \omega^2 u \quad (5)$$

In all simulation studies, the initial values of the plant  $y(0)$ ,  $y'(0)$ , and  $y''(0)$  are zero.

#### Simulation 1 (Step Control)

In order to obtain a small overshoot in positive step control, we use the modification of the integral-square-error criterion

$$\mathbf{J} = \begin{cases} \int_0^{t_0} 100.0 e^2 dt & e > 0 \\ \int_0^{t_0} e^2 dt & e \leq 0 \end{cases} \quad (6)$$

Fig.4 shows the step response to the nonlinear plant with  $\omega = 1.0$  and  $\xi = 1.0$ . Since there is a damping in

this plant to dissipate energy, the parameter  $K_d$  of the D controller is chosen to be zero. Using neural network, we get the parameter vector  $\mathbf{k}^* = (0.472476, 0.431155, 0.410438)$ . It can be observed that the time response exhibits a very small maximum overshoot  $M_p \approx 0.0$  and a fast settling time  $t_s = 0.72$  s.

Now, we set  $\xi = -2.0$ . In this case, there is no damping in the plant to dissipate energy. For dealing with this problem, we add the D controller with  $K_d = -3.5$  (notice that the choice of  $K_d$  is not strict) to dissipate the internal energy of the plant. The optimum membership functions with  $\mathbf{k}^* = (0.485403, 0.474852, 0.463350)$  are determined using neural network in self-organizing process. Fig.5 shows the time response with  $M_p \approx 0.0$  and  $t_s = 0.78$  s.

### Simulation 2 (tracking control):

In tracking control, the standard integral-square-error criterion is used to describe a control performance. Fig.6 shows the sinusoidal tracking response to the plant with  $K_d = 0.0$ ,  $\omega = 1.0$  and  $\xi = 1.0$ . Using neural network, we get the parameter vector  $\mathbf{k}^* = (0.404975, 0.344822, 0.268896)$ . Fig.6 shows the time response with a small tracking error in the sinusoidal tracking control.

Fig.7 shows the ramp tracking response to the plant ( $\omega = 1.0$ ,  $\xi = 1.0$ ,  $K_d = 0.0$ ). The optimum membership functions with  $\mathbf{k}^* = (0.5, 0.55451, 0.448028)$  are found. The simulation result exhibits a small tracking error in the sinusoidal tracking control.

Fig.8 shows the ramp tracking response to the plant ( $K_d = 0.0$ ,  $\omega = 1.0$ ,  $\xi = 1.0$ ). in the presence of the disturbance  $y_{dis} = 0.2 \sin(1.1t)$  in Fig.9. In spite of the disturbance, the tracking error still is small using the membership functions in Fig.7.

## 5 Conclusions

This paper presents a new fuzzy SOC strategy, which optimizes membership functions using neural network according to previous responses. This approach is suitable for systems in the absence of analytical models. The simulation results show the effectiveness and robustness of the proposed method.

The SOC control scheme consists a FL controller and a D controller. The FL controller is used to improve transient behaviors (e.g., small maximum overshoots and a fast settling time). If the FL controller can not stabilize a plant under the strongest negative feedback control, the D controller is added to solve this problem. This strategy may avoid dealing with stability analysis of nonlinear plants controlled by the SOC control scheme.

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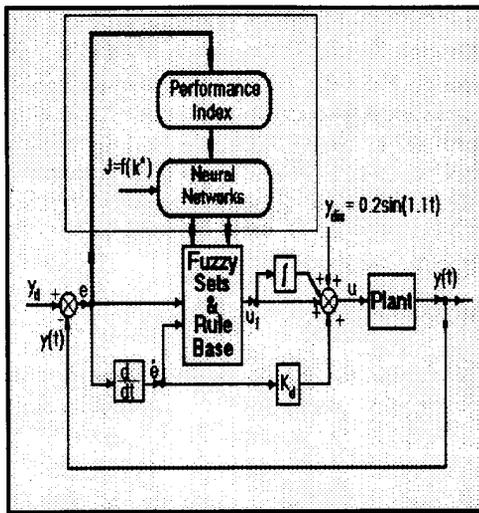


Fig. 1: Control scheme for SOC algorithm

$\frac{c}{e}$	NB	NS	ZO	PS	PB
NB	PB	PB	PB	PS	ZO
NS	PB	PB	PS	ZO	NS
ZO	PB	PS	ZO	NS	NB
PS	PS	ZO	NS	NB	NB
PB	ZO	NS	NB	NB	NB

Table 1: Fuzzy rule base

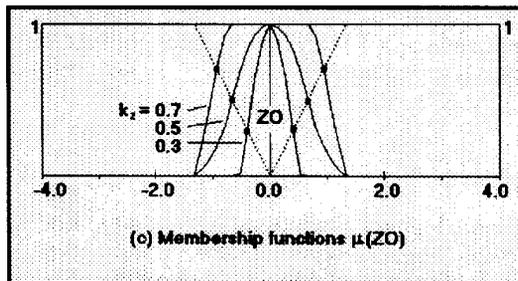
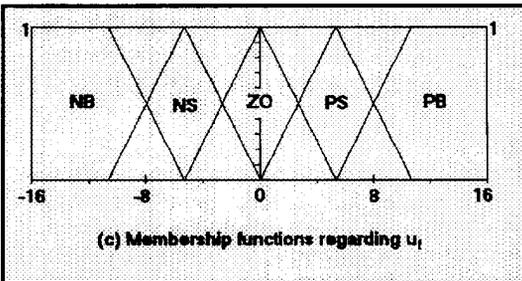
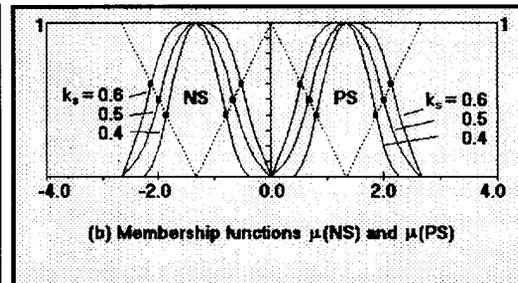
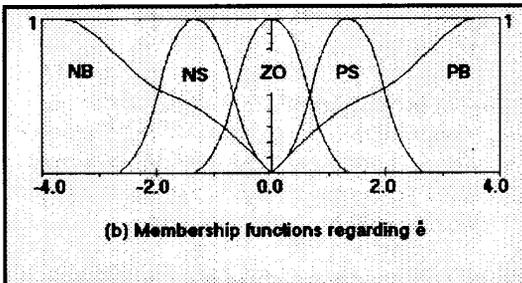
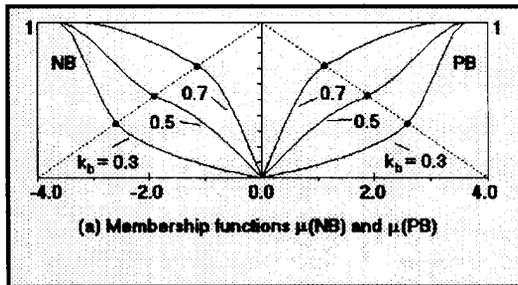
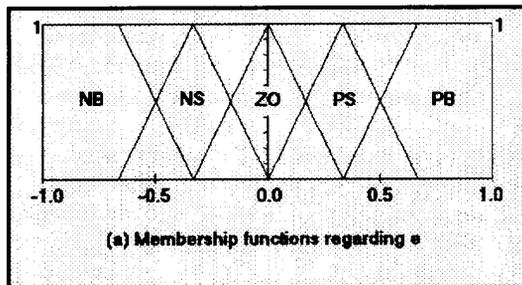


Fig. 2: Membership functions of linguistic variables

Fig. 3: Changes of membership functions' shapes

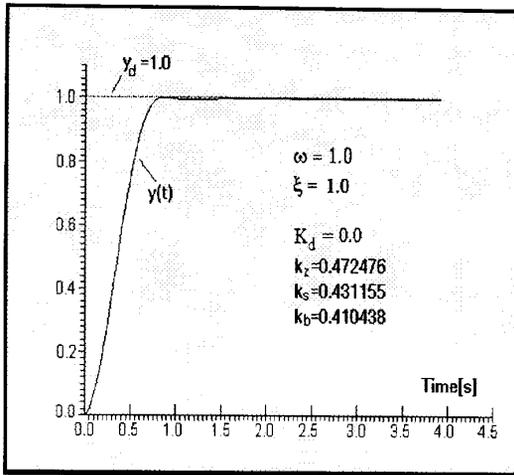


Fig. 4: Time response of a stable plant

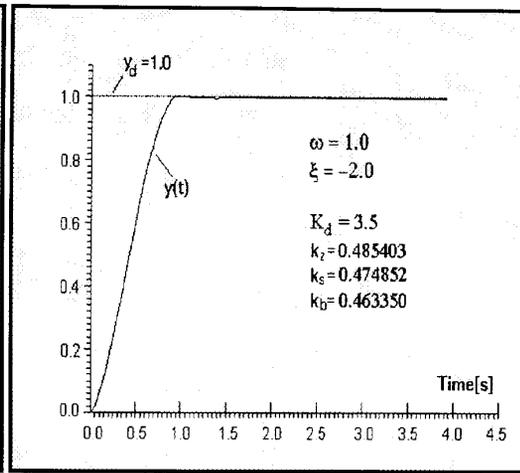


Fig. 5: Time response of an unstable plant

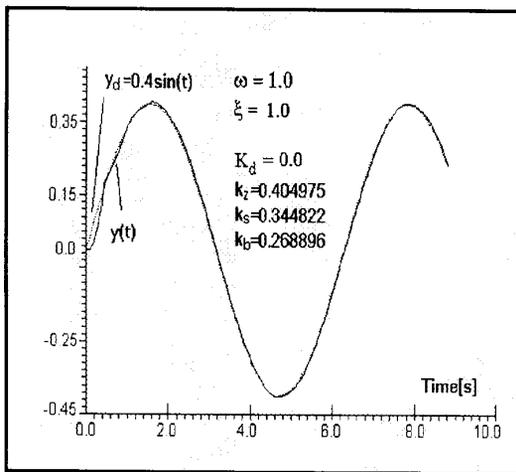


Fig. 6: Time response of sinusoidal tracking

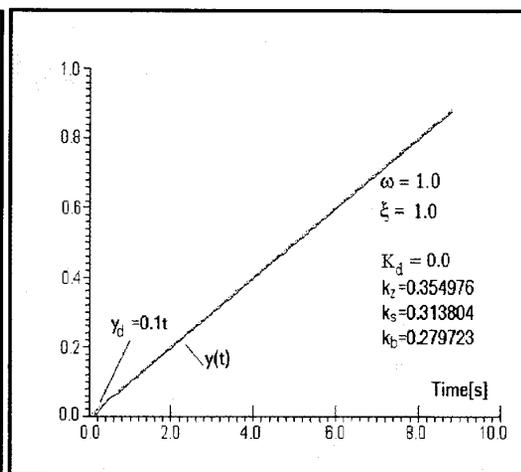


Fig. 7: Time response of ramp tracking

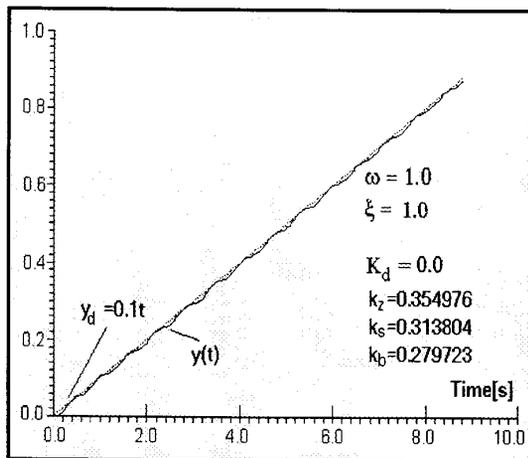


Fig. 8: Time response of ramp tracking in the presence of a disturbance

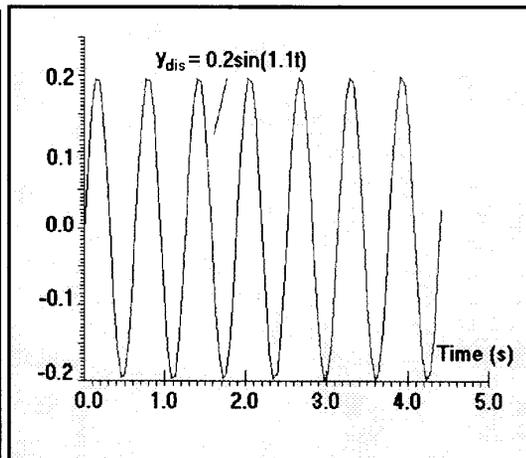


Fig. 9: A disturbance signal