1. Using the equation
\[ \cos(x + y) = \cos x \cos y - \sin x \sin y \]
derive the equation:
\[ \cos(x - y) = \cos x \cos y + \sin x \sin y \]

2. Using the equations
\[ \cos(x + y) = \cos x \cos y - \sin x \sin y \]
\[ \sin(x + y) = \sin x \cos y + \cos x \sin y \]
derive the equation:
\[ \tan(x + y) = \frac{\tan x + \tan y}{1 - \tan x \tan y} \]

3. Let \( f(x) = 3 \sin(2\pi x + \pi/6) \). Rewrite \( f(x) \) as a linear combination of \( \sin 2\pi x \) and \( \cos 2\pi x \) (i.e. find constants \( A \) and \( B \) such that \( f(x) = A \sin 2\pi x + B \cos 2\pi x \)).

4. Is it possible to find the exact value of the sine and cosine of 7.5° using trigonometric identities? Why or why not?

5. Let \( z = \tan(x/2) \). Show that
\[ \cos x = \frac{1 - z^2}{1 + z^2} \quad \text{and} \quad \sin x = \frac{2z}{1 + z^2} \]

6. Why is \( e^{A+B} = e^A e^B \)? Let \( i \) be a (non-real) number such that \( i^2 = -1 \). If \( x \) is a real number then \( ix \) isn’t a real number, but the Swiss mathematician Euler said that one can still define \( e^{ix} \) as
\[ e^{ix} = \cos x + i \sin x \]
Using Euler’s definition, compute \( e^{i(x+y)} \). Using Euler’s definition, compute \( e^{ix} \) and \( e^{iy} \) separately then multiply them together (using the fact that \( i^2 = -1 \)). Equate the two quantities and what do you conclude (think in terms of the addition formulas)?