MATH 201 – LAB 5
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The chain rule tells us how to compute the derivative of a composition of two functions $f$ and $g$ in term of the individual derivatives $f'$ and $g'$; specifically

$$(f \circ g)'(x) = f'(g(x))g'(x)$$

Identify $f$ and $g$ in the following compositions and apply the chain rule to find the derivative of the composition with respect to $x$:

1. $3(x^2 + 3x + 5)^8$.
2. $\sqrt{x^2 + 8x + 3}$.
3. $e^{5x}$.
4. $e^{\sin x}$.
5. $\sin(x \cos x)$.
6. $\sin(x \cos x)/(x^2 + 1)$.

7. In many cases the rate of change of some quantity is proportional to the amount of that quantity, i.e.

$$f'(x) = kf(x)$$

where $k$ is some constant. Show that a composition of the exponential function $e^x$ and a carefully chosen linear function will solve this differential equation.

8. In other cases the second derivative of some quantity is proportional to minus the amount of that quantity, i.e.

$$f''(x) = -kf(x)$$
where $k > 0$. Show that a composition of a trigonometric function and a carefully chosen linear function will solve this differential equation.

9. The surface area $A$ and volume $V$ of a sphere of radius $r$ satisfy

$$A = 4\pi r^2 \quad \text{and} \quad V = \frac{4}{3} \pi r^3$$

Show that the instantaneous rate of change of volume with respect to the radius $dV/dr$ is equal to the current surface area $A$. Is this reasonable?

10. By comparison, the surface area $A$ and volume $V$ of a cube of side $s$ satisfy

$$A = 6s^2 \quad \text{and} \quad V = s^3$$

How is the instantaneous rate of change of volume with respect to the length of the side $s$ related to the current surface area $A$? Explain the difference.