PARTIAL DIFFERENTIAL EQUATIONS
MATHEMATICS 402 - FALL 2002

INSTRUCTOR: Marc Thomas

DESCRIPTION: This course is a standard introduction to the theory and application of parabolic, hyperbolic, and elliptic partial differential equations. We will also cover the necessary mathematical tools, such as transforms, complex numbers, and the concept of an orthogonal family of functions. We will try to cover the following set of topics:

I. Fourier Series and Integrals
   a. Periodic functions and functions on the circle, trigonometric series.
   b. Fourier series, Euler formulas, Dirichlet’s kernel, Complex Fourier series.
   c. Even and odd functions, functions with arbitrary period.
   d. Half range expansions.
   e. Determination of Fourier coefficients, the problem of uniform/pointwise convergence, Dirichlet’s kernel and approximation.
   f. The Fourier integral, (non-periodic) functions on $\mathbb{R}$.

II. Partial Differential Equations
   a. General framework, initial conditions and boundary conditions for a partial differential equation.
   c. The (parabolic) one-dimensional heat equation $u_t = \alpha^2 u_{xx}$.
   d. The (hyperbolic) one-dimensional wave equation $u_{tt} = \alpha^2 u_{xx}$ and the transmission line equation $v_{xx} = CLv_{tt} + (CR + GL)v_t + GRv$.
   e. The two-dimensional wave equation $u_{tt} = \alpha^2 (u_{xx} + u_{yy})$, a square membrane, a circular membrane, polar coordinates, and Bessel’s equation $u_{tt} = \alpha^2 (u_{rr} + u_r/r + u_{\theta\theta}/r^2)$.
   f. Laplace’s (elliptic) equation $\nabla^2 u = 0$ and potential, polar and cylindrical forms, the Poisson integral:

   \[
   \frac{1}{2\pi} \int_0^{2\pi} f(\alpha) \left( \frac{1 - r^2}{1 - 2r \cos(\theta - \alpha) + r^2} \right) d\alpha
   \]

   g. Spherical coordinates and Legendre’s equation.

III. Other important PDE’s and related topics
   a. Sturm-Liouville problems, the associated orthogonal families, and their pertinence to the theory of PDE’s.
   b. The Cauchy-Riemann equations $u_x = v_y$ and $u_y = -v_x$.
   c. Analytic functions of a complex variable.

GRADING: Two midterms will be given, each worth 30%. I do not give make-up midterms; for an excused absence I count the other grades proportionately higher. One final exam, comprehensive but emphasizing the later material will be given. This is mandatory and is worth 30%. Homework and lab work together are worth 10%. Since the desire is that the homework and lab work be a learning experience, these assignments will be graded on a good/satisfactory/unsatisfactory basis. We are required to remind students that the last day to drop is November 12th.