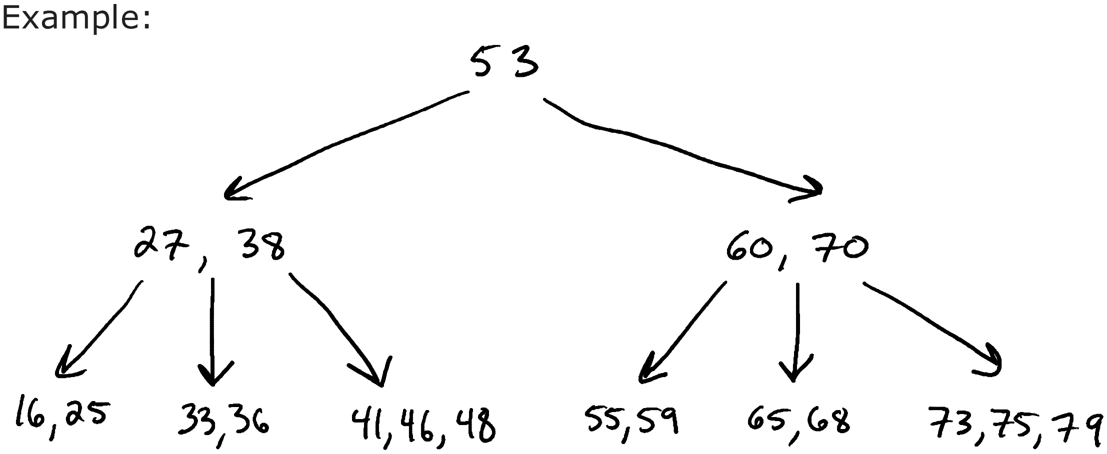
Other Trees

Friday, November 09, 2007
2:12 PM

15.3 2-3-4 Trees & Other Trees

2-3-4 Trees
- extend BST to have more then 2 children
- need to have different relational check than just less-than & greater-then
  - will allow more than 2 search paths
- m-node tree has m children
  - stores valves k1 to k(m-1)
  - children T1 to Tm
  - check value v as follows:
    - v < k1 go to T1
    - k1 <= v < k2 go to T2
    - k2 <= v < k3 go to T3
    - etc until:
    - k(m-1) <= v go to Tm
  - 2-3-4 tree allows m = 2, 3 or 4
  - BST is m = 2 only
- 2-3-4 ADT
  - Data: A tree where
    - 1 each node stores 1 to 3 values
    - 2. each non-leaf node is an m-node w/ m = 2, 3 or 4
    - 3. all leaves are on the same level
  - Operations
    - create empty
    - check empty
    - search for an item
    - insert an item; maintain 2-3-4 property
    - delete an item; maintain 2-3-4 property
- Example:



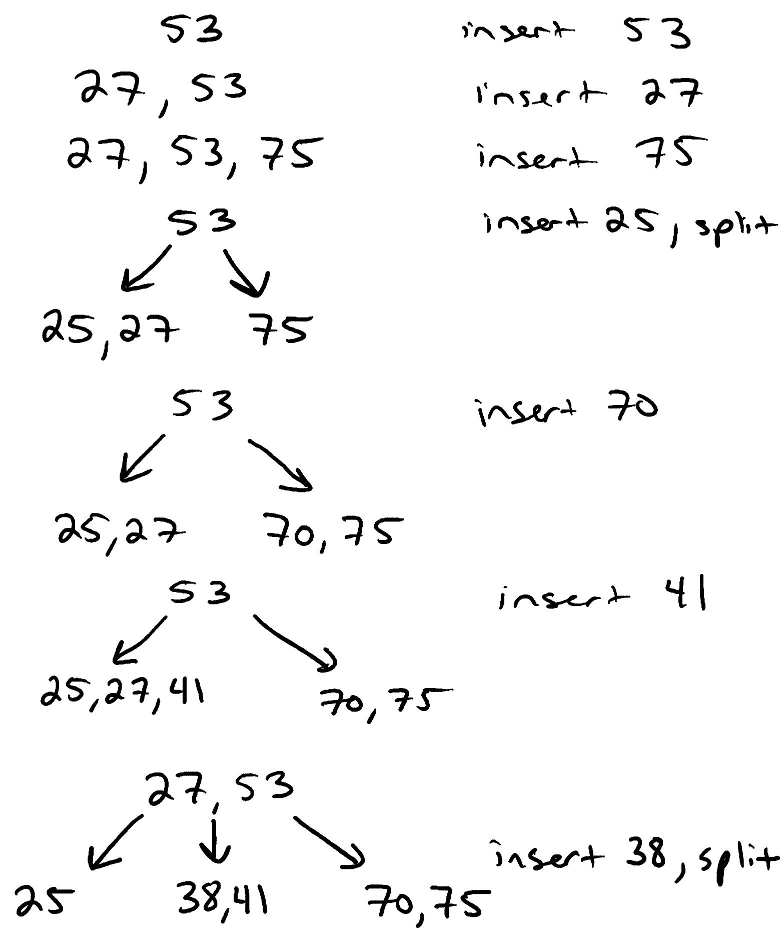
Inserting an item
- Must maintain property 3 which keeps the tree balanced

Pseudocode

```
if tree is empty
    create 2-node w/ item & make it root
else
    find the leaf node where item belongs
    if leaf contains < 3 valves
        add item to leaf
    else
        split leaf into two nodes
            median of 4 values used as "root" for this subtree
            all values < median go into one node (1 or 2 values)
            all valves > median go into the other (1 or 2 values)
        set node to original leaf
        set parent to node
        set split to true
        while split is true
            if parent is NULL
                create new 2-node w/ median
                make two new nodes children of 2-node
                set root to 2-node
            else if parent has < 3 values
                add median to parent values
                replace node w/ two new nodes
                set split to false
            else
                split parent into two nodes using same method as
                above
                set node to parent
                set parent to parent's parent
```

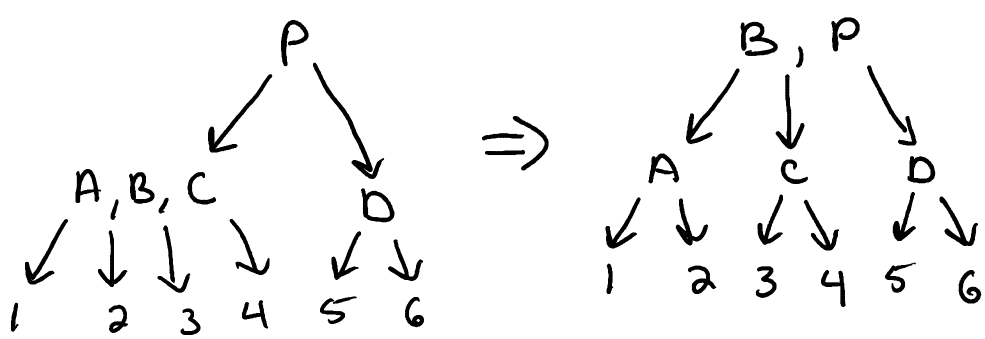
Example:

insert 53, 27, 75, 25, 70, 41, 38



An alternative to splitting up w/ the while loop is to split all 4-node to two 2-nodes while searching for leaf to insert the item

- this is called top-down insertion
- eliminates while loop
- faster since only visit each node once



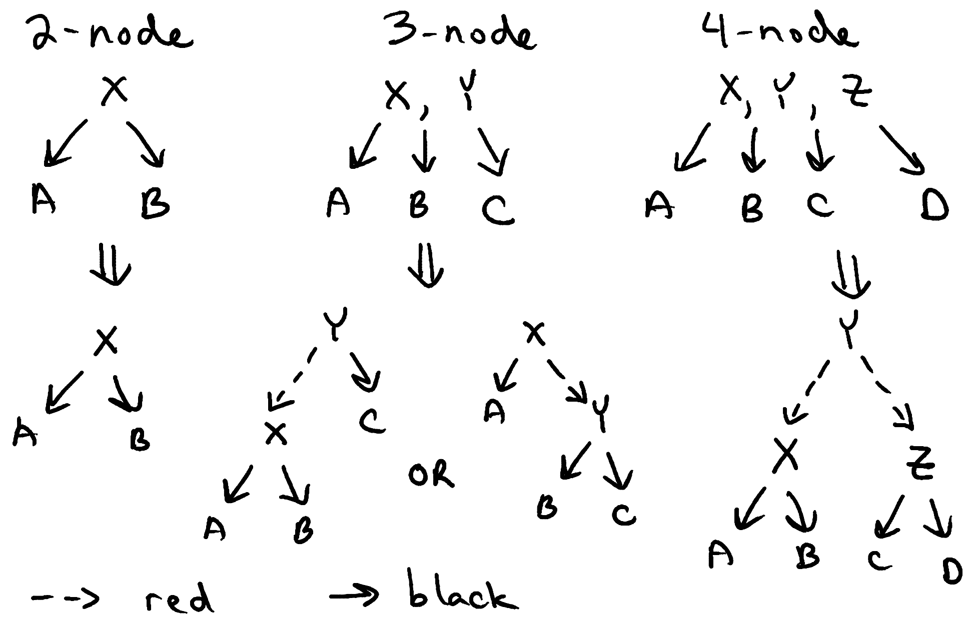
Data Storage

- Simple implementation
  - array of 3 for values
  - array of 4 node pointers for children

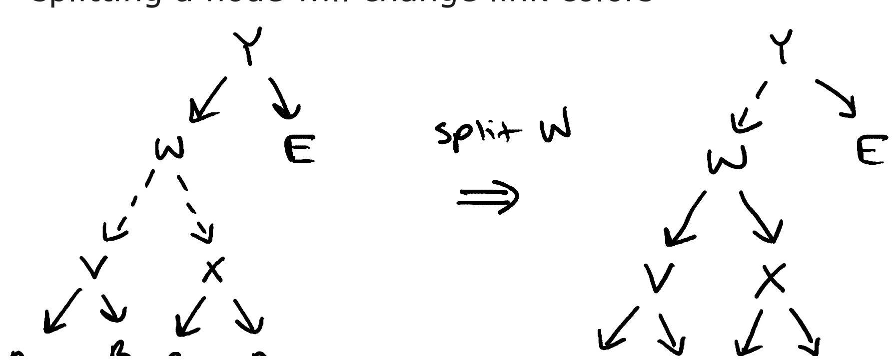
- Simple implementation is inefficient
  - always allocates space for a 4-node
  - wasted memory for 2-node & 3-node
  - approximately 75% of memory is wasted
- can use BST to represent any tree but BST will not stay balanced
  - red-black trees can also be used to represent 2-3-4 trees

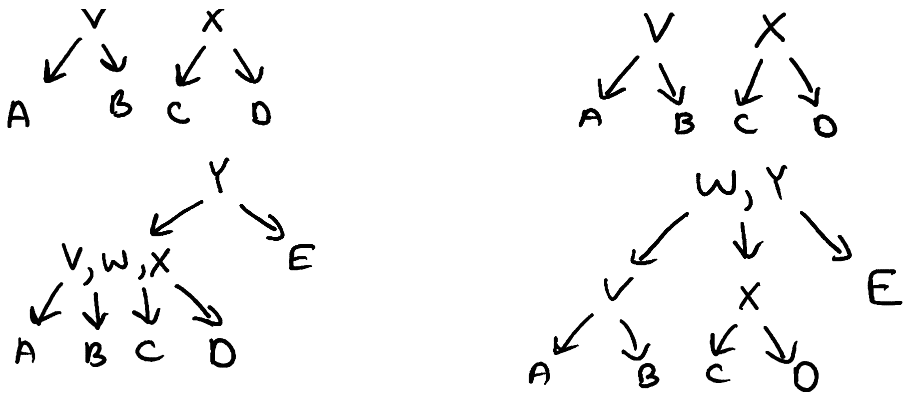
Red-Black trees

- BST tree w/ colored links (red & bled)
- kept balanced using AVL-like rotations
- maintains the following properties:
  1. Each path from the root to a leaf node has the same number of black links
  2. No path from the root to a leaf has two or more consecutive red links
- Note: this is one definition. An alternative definition is:
  - each node is colored red or black
  - the root node is black
  - if a node is red, its children must be black
  - every path from a node to a NULL "leaf " must contain the same number of black nodes
- To represent 2-3-4 tree as red-black
  - make the link black if it is an actual link in the 2-3-4 tree
  - make the link red if it connects parts of the same node in 2-3-4 tree

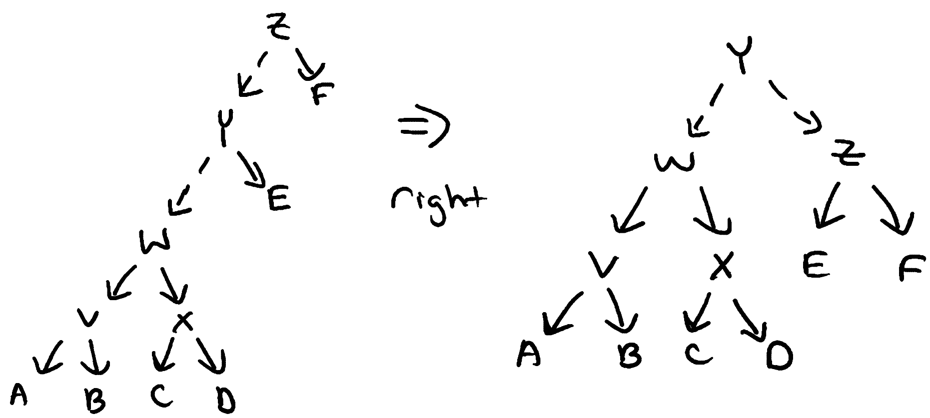


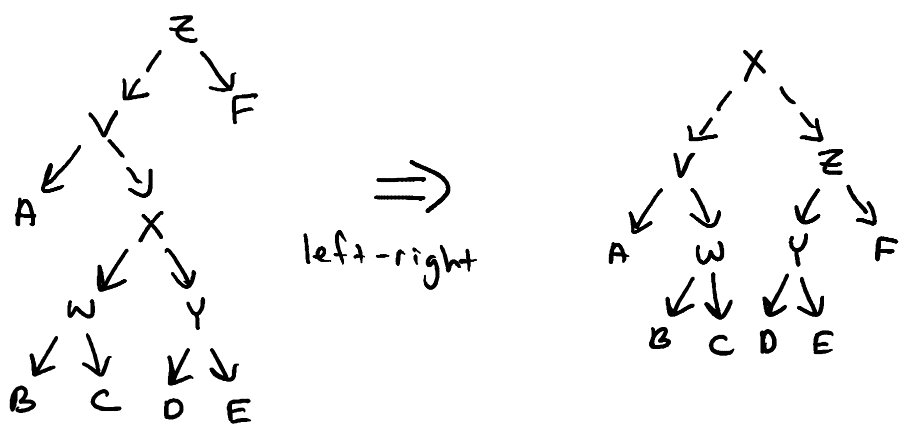
splitting a node will change link colors





the split may cause two consecutive red links
for example, there could be a red link to Y
use AVL rotations to remove consecutive red links





B-Trees
- book's definition is weak, using another
- use m-node concept like 2-3-4 tree
- can use external storage for data
  - data items are stored in leaves, which can be on disk
- Definition
  - all data is stored in leaves
  - non-leaf nodes store keys to data on disk
  - the root is either a leaf or has between 2 and M children
  - the non-leaf nodes have ceiling(M/2) to M children
  - all leaves have ceiling(L/2) to L data items

choose L & M based on amount of data to be stored

- affects number of nodes needed to index the data
- nodes are in main memory, so want to choose values that will allow all nodes to be stored
- L & M can be the same
    - 2-3-4 tree has L=M= 4

Example: