This lab considers some statistical estimators. Assume that a random sample has been taken from some population and the sequence of numbers obtained are \( \{x_i\}_{i=1}^n \). Then we can define the statistical estimators:

\[
\text{mean } = \bar{x} = \frac{1}{n} \sum_{i=1}^{n} x_i
\]

and

\[
\text{variance } = \sigma^2 = \frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})^2
\]

1. The formula above for the variance has the deficiency that one has to go through the numbers twice, the first time to compute the mean and the second time to compute the variance. Rewrite the second formula to show that

\[
\text{variance } = \sigma^2 = \frac{1}{n-1} \left( \sum_{i=1}^{n} x_i^2 - \frac{1}{n} \left( \sum_{i=1}^{n} x_i \right)^2 \right)
\]

so that it is possible to get both estimates in one pass just by keeping track of the number of numbers, the sum of the numbers, and the sum of the square of the numbers.

2. Look at the sample program `summation.c` which takes a list of decimal numbers, ending with `end`, and prints out the sum of the numbers.

**Assignment** There are two parts to this assignment. The first part is to

Write a program, similar to the sample program `summation.c` which will take a list of decimal numbers, ending with `end`, and print out the estimates of the mean \( \bar{x} \) and the variance \( \sigma^2 \). Email me the path to your program in plain text. For example, you might say that the path to your solution is

```
/usr/stu/yourusername/cs305/lab1.c
```

This is all I need. **Do not email attachments of code to me.**

The second part is to answer the following questions. You can either turn in your answers on paper or email me your answers in a plain text file.

1. Derive the Taylor series for \( f(x) = (1 + x)^r \) expanded about \( x_0 = 0 \). Assume that \( r > 0 \in \mathbb{R} \). Where does the series converge if \( r \) is a positive integer? Where does the series converge if \( r = 3/2 \)?

2. Derive the Taylor series for the polynomial \( p(x) = x^3 - 2x^2 + 4x - 1 \) expanded about \( x_0 = 2 \). Use this to write a formula for \( f(2 + h) \).

3. Convert the decimal number 31.1 to binary. Convert the binary number 100110.1012 to decimal.
4. Explain why a rational number has a repeating expansion in either decimal or binary.

5. Suppose that we declare
   
   ```csharp
   float x = 0.125;
   double y = -0.875;
   ```

   Find the IEEE 32-bit representation of \( x \) and the IEEE 64-bit representation of \( y \).

6. What real numbers are represented by the 32-bit machine words (in IEEE floating point representation) below?
   
   0 00000001 00000000000000000000000
   0 10000001 01110000000000000000000

7. If \( 2^{10} \) is represented by a `double` what is the closest machine `double` to the right of it? Recall that the machine epsilon (for a `double`) is \( 2^{-52} \).

8. It is common advice that if a long list of real numbers is to be added by a computer program, then the numbers should be sorted first and then added starting from smallest and working to largest. Why do you think that this advice is given?

9. Give an example of machine arithmetic on three real numbers where \((a + b) + c\) is not equal to \( a + (b + c) \).

10. A programmer is going to use single precision `float` variables to record the elevation of points on the earth measured in centimeters from the center of the earth (e.g. Bakersfield will be at average elevation 1,287,475,200 using these units). He wants to record variations in elevation within a given city and compute a standard deviation. Is this a good plan? Why or why not? If not, what changes would you recommend?