CMPS 2120 **Name (Print):** ____________________

Fall 2019

Practice Exam 1

Date: **Time Limit: 120 Minutes**

This practice exam contains 9 pages (including this cover page) and 8 problems. Check to see if any pages are missing. Enter all requested information on the top of this page.

Please treat this as if it were a real exam. You may *not* use your books, notes, or any calculator with a computer algebraic system (CAS) on this exam.

You are required to show your work on each problem on this exam. The following rules apply:

- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.

1. Using these propositions:

P: Paul will go hiking

W: The weather is sunny

R: Paul is feeling ill

Express each of the following compound propositions in English.

a) $\neg R$

b) $W \Longrightarrow P$

c) $W \wedge R$

d) $\neg P \Longrightarrow \neg W$

e) $R \Longleftrightarrow P$

f) $(W \wedge \neg R) \Longrightarrow P$

g) $(P \vee R) \wedge \neg(P \wedge R)$

h) $\neg(\neg P \vee W) \Longrightarrow \neg R$

2. Determine whether each of the following is a tautology or a contradiction, or neither.

a) $p \iff p \land (p \lor q)$

b) $[p \implies (q \land r)] \implies [r \implies (p \implies q)]$

c) $(p \land \neg p) \implies (q \lor \neg q)$

3. For a and b, State the converse, contrapositive, and inverse of each of these conditional statements. For c, address the question.

   Conditional: $p \implies q$

   Converse: $q \implies p$

   Contrapositive: $\neg q \implies \neg p$

   Inverse: $\neg p \implies \neg q$

   a) A grade point average of 3.0 is sufficient to graduate with honors.

   b) A sequence S is bounded whenever S is convergent.

   c) Prove that a conditional statement and its inverse are equivalent.

4. Which of the following statements is true? If false, give an example of why it's false. (the universe is all positive integers)

   a) $(\exists x \forall y)[2x - y = 0]$

   b) $(\exists y \forall x)[2x - y = 0]$

   c) $\forall x(x < 10 \implies \forall y(y < x \implies y < 9))$

   d) $\exists y \exists z(y + z = 100)$

   e) $\forall x \exists y(y > x \land \exists z(y + z = 100))$

5. Write the following statements in logical forms:

   a) Anyone who has forgiven at least one person is a saint.
      $S(x)$: x is a saint.
      $F(x,y)$: x forgives y.

   b) Nobody in the calculus class is smarter than everybody in the discrete math class.
      $C(x)$: x is in calculus.
      $D(x)$: x in discrete math class.
      $S(x,y)$: x is smarter than y.

c) Everyone likes Mary, except Mary herself.

$L(x,y)$: x likes y.

$M(x)$: x is Mary.

d) Jane saw a police officer, and Roger saw one too.

$S(x,y)$: x saw y.

$P(x)$: x is a police officer.

6. Prove the following statements:

a) Suppose x is a positive integer. If 8 does not divide $x^2 - 1$, then x is even.

b) Suppose a and b are positive integers. If ab is odd, then both a and b are odd.

c) If $x^2 \leq 1$, then $x^2 - 7x > -10$.

d) Prove that 3 divides $(n^3 - n)$, $\forall\, n \geq 1$

7. Answer the following regarding sets.

   a) Let $A = \{\emptyset, a, b\}$. Find the power set of A: $P(A)$

   b) Given: $A = \{1,2,3,4,5,6\}$ and $B = \{a,b,c,d\}$. Give an example of a function from A to B that is neither one-to-one nor onto and explain why it meets these conditions.

8. Draw the following set relations in a Venn Diagram. Shade the required regions.

a) $A \cup (B \cap C)$

b) $(A \cap B) \cup C$

c) $A \cup \overline{(B \cap C)}$