Design of a Hybrid Fuzzy Logic Proportional Plus Conventional Integral-Derivative Controller

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***Abstract*— This paper presents approaches to the design of a hybrid fuzzy logic proportional plus conventional integral-derivative (fuzzy P+ID) controller in an incremental form. This controller is constructed by using an incremental fuzzy logic controller in place of the proportional term in a conventional PID controller. By using the bounded-input/bounded-output "small gain theorem," the sufficient condition for stability of this controller is derived. Based on the condition, we modify the Ziegler and Nichols' approach to design the fuzzy P+ID controller. In this case, the stability of a system remains unchanged after the PID controller is replaced by the fuzzy P+ID controller without modifying the original controller parameters. When a plant can be described by any modeling method, the fuzzy P+ID controller can be determined by an optimization technique. Finally, this controller is used to control a nonlinear system. Numerical simulation results demonstrate the effectiveness of the fuzzy P+ID controller in comparison with the conventional PID controller, especially when the controlled object operates under uncertainty or in the presence of a disturbance.**

***Index Terms*—Fuzzy logic, fuzzy P+ID controller, hybrid system, stability.**

I. Introduction

It is well known that up until now, a conventional proportional integral-derivative (PID)-type controller is most widely used in industry due to its simple control structure, ease of design, and inexpensive cost. However, the PID-type controller cannot yield a good control performance if a controlled object is highly nonlinear and uncertain. Another type of controller based on fuzzy logic [1] is being increasingly applied to many systems with nonlinearity and uncertainty [2]. Especially, the most successful fuzzy logic controllers applied into industrial plants are designed by control engineers. Unfortunately, defining membership functions of linguistic variables and formulating fuzzy rules by manual operation is time consuming work [3]–[6]. Besides, a few of stability analysis for the real applied fuzzy logic controllers are reported so that many people worry about their reliability.

One of the motivations for this study is trying to answer these questions by proposing a hybrid fuzzy P+ID controller. This controller is constructed by replacing the proportional term in the conventional PID controller with an incremental fuzzy logic controller. For implementing the fuzzy P+ID controller, only one additional parameter has to be adjusted. Consequently, the manual tuning time of the controller can be greatly reduced in comparison with a traditional fuzzy logic controller. Another motivation stems from our application of fuzzy logic controllers in industry. The control results on a manipulator and stoker-fired boilers [6], [7] show the effectiveness and the robustness of the hybrid fuzzy logic plus conventional controllers. In fact, a lot of industrial plants have been controlled by a PID-type controller. Many control engineers hope that a fuzzy logic controller should improve control performance yielded by the PID-type controller even without deteriorating the system stability.

Recently, fuzzy-logic and conventional-techniques are combined to design fuzzy logic controller, such as [5]–[16]. Some of the important research results among them are addressed in [10] and [11]. First, the formulations of the fuzzy proportional derivative (PD) and fuzzy PI+fuzzy derivative (D) control systems are derived based on the "center of mass" approach; and second, "small gain theorem" is used to analyze their bounded-input/bounded output stability. In this paper, this methodology is used to formulate the proposed fuzzy P+ID controller and analyze its stability. The main idea of the study is to use a conventional D controller to stabilize a controlled object and the fuzzy proportional (P) controller to improve control performance. According to the stability condition, we modify the Ziegler and Nichols' approach to design of the fuzzy P+ID controller since this approach is still widely used in industrial control of a plant with unknown structure or with nonlinear dynamics. When the controlled object is described by any modeling method, such as behavior modeling [17], the fuzzy P+ID controller can be determined by an optimization approach, such as genetic algorithms.

In comparison with the exiting fuzzy PID controllers, the proposed fuzzy P+ID controller combines the advantages of a fuzzy logic controller and a conventional controller. The fuzzy P term plays an important role in improving an overshoot and a rise time response. The conventional integral (I) term reduces a steady-state error, and the conventional D is responsible for the stability of the system and the flatness of the response. Furthermore, this controller has the following features.

1) Since it has only one additional parameter to be adjusted based on the original PID controller it is easy to design.
2) The fuzzy P+ID controller keeps the simple structure of the PID controller. It is not necessary to modify any hardware parts of the original control system for implementation.

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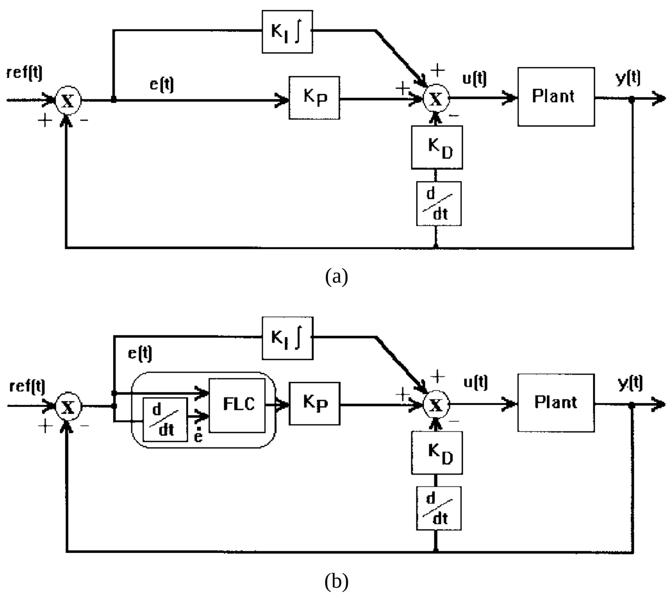


Fig. 1. Control schemes. (a) PID-type controller. (b) Fuzzy P+ID controller.

3) The sufficient stability condition shows that the same stability remains unchanged if the original PID controller is replaced by the fuzzy P+ID controller.

This paper is organized as follows: In the next section, we present the fuzzy P+ID control scheme. In Section III, we formulate an incremental fuzzy logic controller. In Section IV, we discuss the sufficient stability condition for the controller. In Section V, we propose approaches to design of the proposed controller. In Section VI, we report simulation results on control of a nonlinear system by the PID-type controller and the fuzzy P+ID controller. Finally, some remarks are given in the conclusion section.

II. Hybrid Fuzzy P+ID Controller Scheme

At present, the PID-type controller is most widely adopted in industrial application due to its simple structure, as shown Fig. 1(a). Its control signal is easily computed by combining proportional integral-derivative terms

$$u(t) = K_{\rm P}e(t) + K_1 \int e(t)\, dt - K_{\rm D}\dot{y}(t) \tag{1}$$

where $K_{\rm P}$, $K_{\rm I}$, and $K_{\rm D}$ are the controller parameters. Its discritized and incremental form is expressed by

$$\Delta u(k) = u(k) - u(k-1) = K_{\rm P}[e(k) - e(k-1)] + K_1 Te(k) - K_{\rm D}\frac{y(k) - 2y(k-1) + y(k-2)}{T}. \tag{2}$$

The reason for the popular use of the PID-type controller is that this controller can be easily designed by adjusting only three controller parameters: $K_{\rm P}$, $K_{\rm I}$, and $K_{\rm D}$. In addition, its control performance can be accepted in many applications. Therefore, we would like to keep the advantages of the PID-type controller while designing a fuzzy logic controller. This idea leads to propose a hybrid fuzzy P+ID controller shown in Fig. 1(b). This hybrid controller uses an incremental fuzzy logic controller in place of the proportional term while the integral and derivative terms keep unchanged

$$\Delta u(k) = u(k) - u(k-1) = K_{\rm P}^{*}\Delta u_f(k) + K_{\rm I}Te(k) - K_{\rm D} \cdot \frac{y(k) - 2y(k-1) + y(k-2)}{T} \tag{3}$$

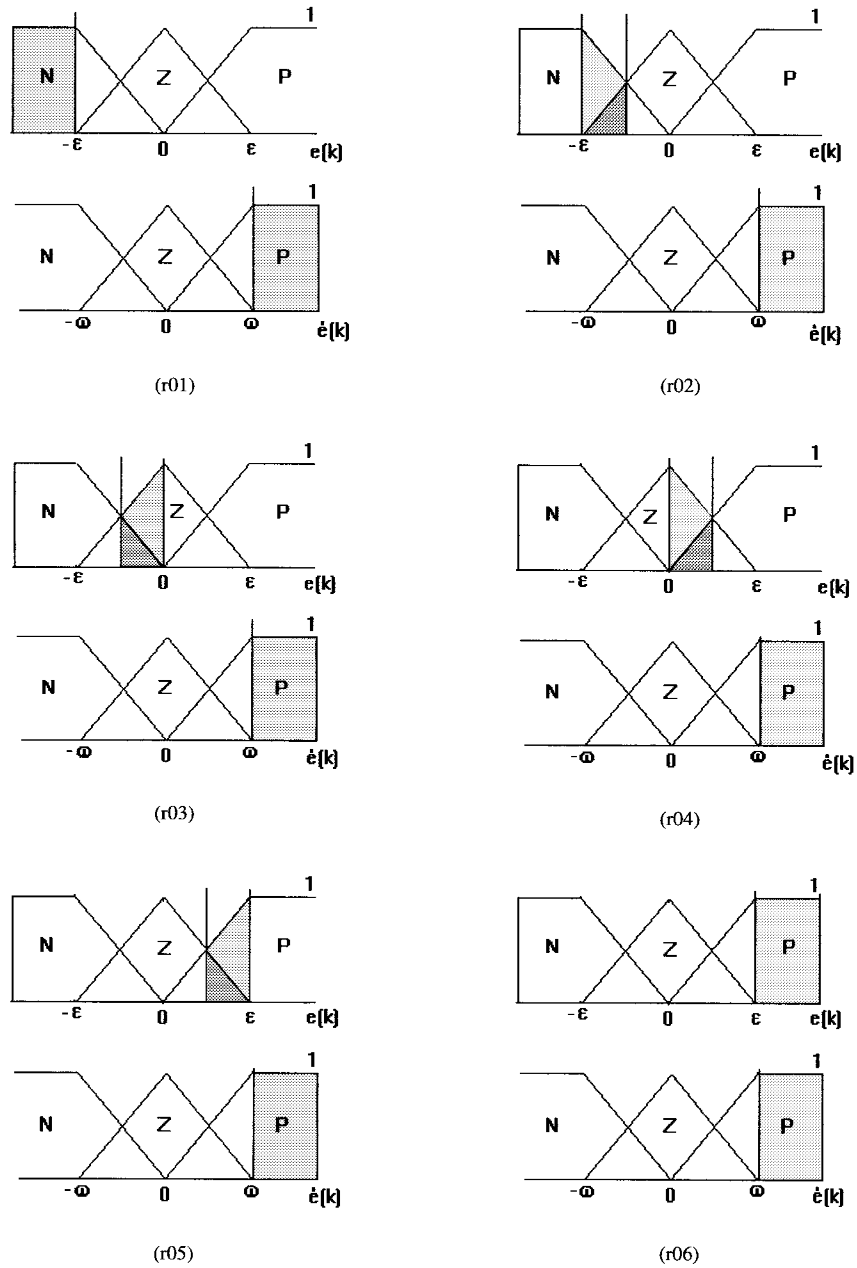
where $K_{\rm I}$ and $K_{\rm D}$ are identical to the conventional PID controller in (2), $\Delta u_f(k)$ is the output of the incremental fuzzy logic controller, and $K_{\rm P}^{*}$ is its proportional coefficient. The relationship between $K_{\rm P}^{*}$ and $K_{\rm P}$ will be discussed in Section IV. The most important part in the fuzzy P+ID controller is the fuzzy P term because it is responsible for improving overshoot and rise time. The conventional I term is mainly responsible for reducing a steady-state error if an actual value is close to a reference signal. The conventional D term is responsible for the stability of the system and for the flatness of the response.

III. Formulation of Incremental Fuzzy Logic Controller

In the fuzzy P+ID controller in (3), its proportional term $K_{\rm P}^{*}\Delta u_f(k)$ is used in place of $K_{\rm P}\Delta e(k)$ in the incremental PID controller, as shown in Fig. 1(b). The incremental fuzzy logic controller is a standard one which has two inputs $e(k)$ and $\dot{e}(k)$ and an output $\Delta u_f(k)$. In this paper, the membership functions of the inputs and the output are defined, as shown in Fig. 2. The parameters ϵ and ω are variables of the membership functions with regard to the fuzzy P controller's inputs $e(k)$ and $\dot{e}(k)$ and we define $\epsilon = \lambda\omega$. In this controller, membership functions (N, Z, P) assigned with linguistic variables are used to fuzzify physical quantities. For inputs $e(k)$ and $\dot{e}(k)$ we have $(e.n, e.z, e.p)$ and $(\dot{e}.n, \dot{e}.z, \dot{e}.p)$. For the output $\Delta u_f(k)$, we have $(o.n, o.z, o.p)$. The fuzzified inputs are inferred to a fuzzy rule base, which is used to characterize the relationship between fuzzy inputs and fuzzy outputs. In this study, the fuzzy rule base of the incremental fuzzy logic controller is fixed, as shown in Table I. It consists of the following nine fuzzy rules:

Rule 1: If $e(k) = N$ and $\dot{e}(k) = P$ then $\Delta u_f(k) = Z$
Rule 2: If $e(k) = Z$ and $\dot{e}(k) = P$ then $\Delta u_f(k) = P$
Rule 3: If $e(k) = P$ and $\dot{e}(k) = P$ then $\Delta u_f(k) = P$
Rule 4: If $e(k) = N$ and $\dot{e}(k) = Z$ then $\Delta u_f(k) = N$
Rule 5: If $e(k) = Z$ and $\dot{e}(k) = Z$ then $\Delta u_f(k) = Z$
Rule 6: If $e(k) = P$ and $\dot{e}(k) = Z$ then $\Delta u_f(k) = P$
Rule 7: If $e(k) = N$ and $\dot{e}(k) = N$ then $\Delta u_f(k) = N$
Rule 8: If $e(k) = Z$ and $\dot{e}(k) = N$ then $\Delta u_f(k) = N$
Rule 9: If $e(k) = P$ and $\dot{e}(k) = N$ then $\Delta u_f(k) = Z$.

The response of each fuzzy rule is weighted according to the degree of membership of its input conditions. The inference engine provides a set of control actions according to fuzzified inputs. The commonly used inference engine is the MAX-MIN method. In the rule base shown in Table I, only Zadeh's logical "AND," which is the MIN operator is used. Since the control actions are described in a fuzzy sense, a defuzzification method is required to transform fuzzy control actions into a crisp output value of the fuzzy logic controller. For the incremental



(a)

Fig. 2. Formulation of the incremental fuzzy logic controller by fuzzy reasoning.

fuzzy logic controller, a widely used defuzzification method is the "center of mass" formula [10], [18]

$$\Delta u_f(k) = \frac{\sum \left\{ \begin{array}{c} \text{membership value of input} \\ \times \text{ output corresponding to the} \\ \text{membership value of input} \end{array} \right\}}{\sum \{\text{membership value of input}\}}. \quad (4)$$

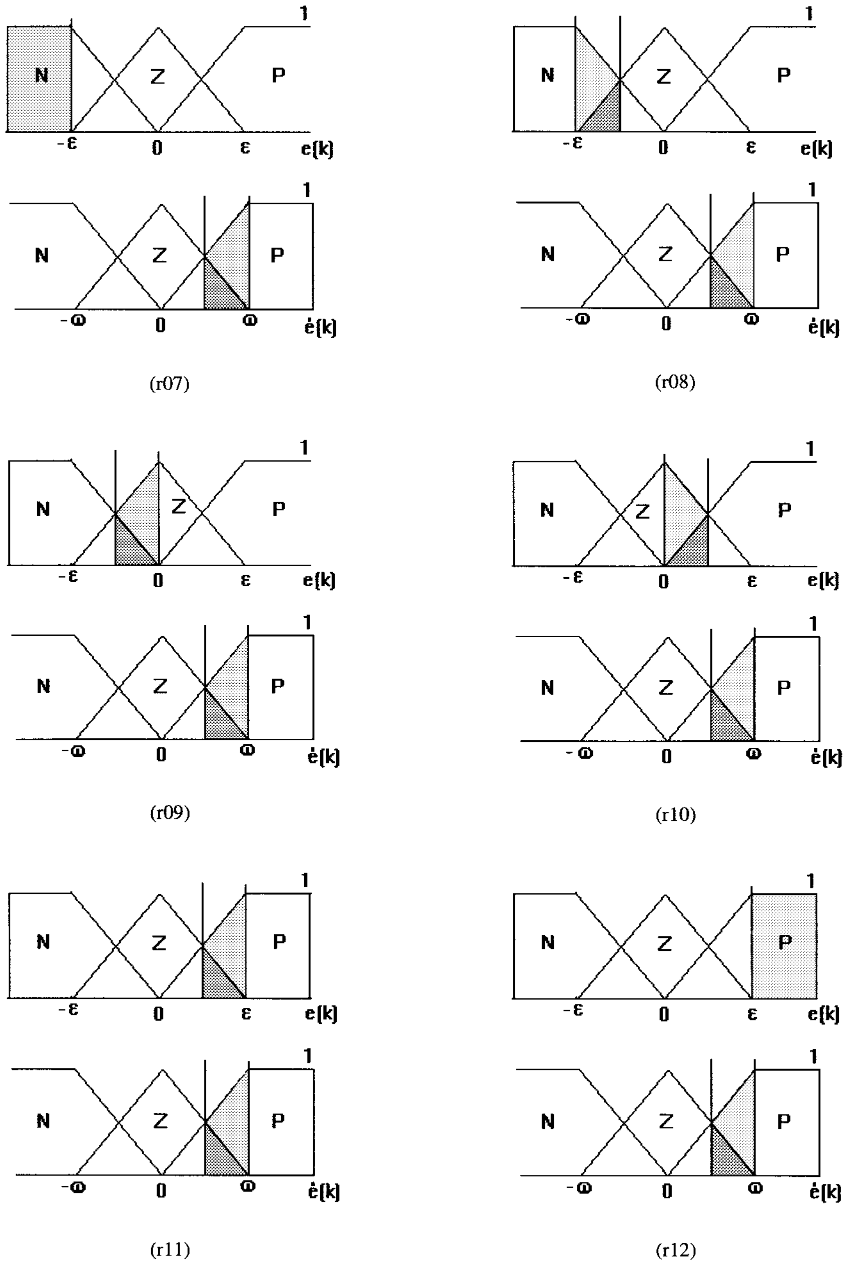
In order to analyze the fuzzy P+ID controller, we derive the formulation of the incremental fuzzy logic controller $\Delta u)f(k) = FLC(e(k), \dot{e}(k))$ as follows. According to the defined inputs' membership functions and the fuzzy rule base, we get 36 combination regions, as shown in Fig. 2. Therefore, the output of the incremental fuzzy logic controller can be expressed by

$$\Delta u_f(k) = \{\Delta u_f^{(1)}(k), \cdot, \Delta u_f^{(i)}(k), \cdots, \Delta u_f^{(36)}(k)\} \quad (5)$$

Since $\Delta u_f(k) = FLC(e(k), \dot{e}(k))$ is an antisymmetrical function, that is $-\Delta u_f(k) = FLC(-e(k), -\dot{e}(k))$, it is easy to obtain the following relationship:

$$\Delta u_f^{(j)}(k) = -\Delta u_f^{(36-j+1)}(k), \qquad j = 1, \cdots, 18. \quad (6)$$

On the basis of (6), we only need to compute $\{\Delta u_f^{(1)}(k), \cdots, \Delta u_f^{(18)}(k)\}$ by fuzzy reasoning. In doing it, we divide the first 18 regions into three categories. Regions in which there is only one rule to be activated are defined as the first type. Only regions 1 and 6 belong to the first type because none of membership functions of $e(k)$ and



(b)

Fig. 2. *(Continued.)* Formulation of the incremental fuzzy logic controller by fuzzy reasoning.

$\dot{e}(k)$ overlaps. Regions in which there are two rules to be activated simultaneously are defined as the second type. In this case, membership functions of $e(k)$ (or $\dot{e}(k)$) overlap each other. By observing Fig. 2 [(r01)–(r18)], regions 2–5, 7, 12, 13, and 18 belong to this type. Regions in which there are four rules to be activated simultaneously are defined as the third type. In this case, membership functions of both $e(k)$ *and* $\dot{e}(k)$ overlap each other. Regions 8–11 and 14–17 belong to the third type.

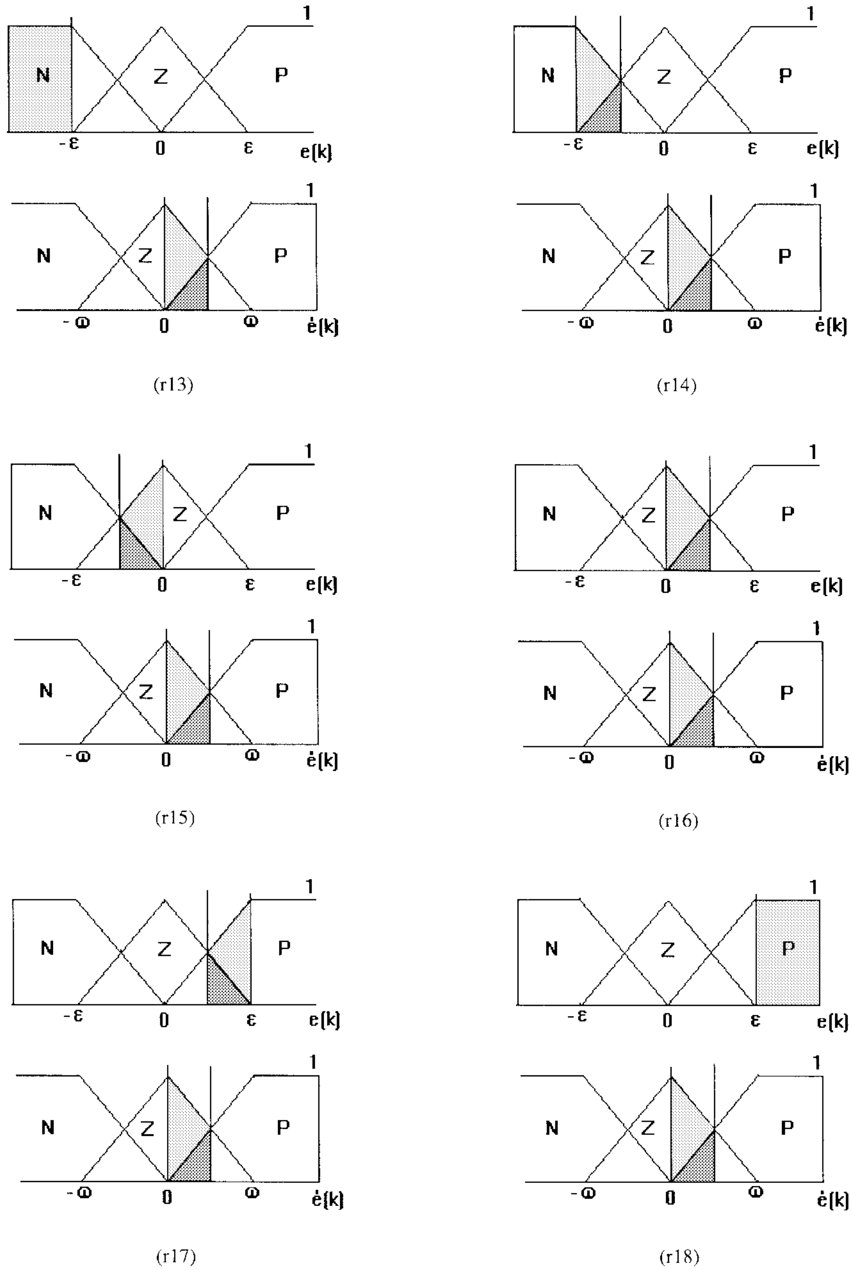
For the first type, we only need to compute $\Delta u_f^{(1)}(k)$ and $\Delta u_f^{(6)}(k)$. In region 1 $(e(k), \leq -\epsilon, \dot{e}(k) \geq \omega)$, only Rule 1 is activated, as shown in Fig. 2 (r01). In this case, we have the input membership functions values $\mu(e.n) = \mu(\dot{e}.p) = 1$ and, thus, $\{\mu(e.n)$ AND $\mu(\dot{e}.p)\} = \min\{1, 1\} = 1$. Since the corresponding output value is $o.z = 0$, using the "center of mass" formula (4) we obtain

$$\Delta u_f^{(1)}(k) = 0 \quad e(k) \leq -\epsilon; \;\; \dot{e}(k) \geq \omega. \tag{7}$$

In region 6 $(e(k) \geq \epsilon, \dot{e}(k) \geq \omega)$, only Rule 3 is activated, as shown in Fig. 2 (r06). It is easy to obtain

$$\Delta u_f^{(6)}(k) = \epsilon \quad e(k) \geq \epsilon; \;\; \dot{e}(k) \geq \omega \tag{8}$$

Now we compute $\Delta u_f^{(j)}(k)$ of the second type as follows. In regions 2 and 3 $(e(k) \in [-\epsilon, 0], \dot{e}(k)n \geq \omega)$, Rules 1 and 2 are simultaneously activated, as shown in Fig. 2 (r02)–(r03). For Rule 1, we have the input membership functions values



(c)

Fig. 2. *(Continued.)* Formulation of the incremental fuzzy logic controller by fuzzy reasoning.

$(\mu(e.n) = -(e(k)/\epsilon) \leq 1\ \mu(\dot{e}.p) =)$ and, thus, $\{\mu(e.n)$ AND $\mu(\dot{e}.p)\} = \min\{-(e(k)/\epsilon), 1\} = -(e(k)/\epsilon)$ with the corresponding output value $o.z = 0$. For Rule 2, we have the input membership functions values $(\mu(e.z) = (e(k) + \epsilon/\epsilon) \leq 1$, and thus $\{\mu(e.z)$ AND $\mu(\dot{e}.p)\} = \min\{(e(k) + \epsilon/\epsilon), 1\} = (e(k) + \epsilon/\epsilon)$ with the corresponding output value $o.p = \epsilon$. By using the "center of mass" formula (4), we obtain

$$\begin{aligned}\Delta u_f^{(2,3)}(k) &= \frac{\mu(e.n) \times o.z + \mu(e.z) \times o.p}{\mu(e.n) + \mu(e.z)} \\ &= \frac{\epsilon(e(k) + \epsilon)}{-e(k) + e(k) + \epsilon} \\ &= e(k) + \epsilon \qquad -\epsilon \leq e(k) \leq 0; \quad \dot{e}(k) \geq \omega. \end{aligned} \tag{9}$$

Similarly, we can obtain

$$\Delta u_f^{(4,5)}(k) = \epsilon \qquad 0 \leq e(k) \leq \epsilon; \quad \dot{e}(k) \geq \omega \tag{10}$$

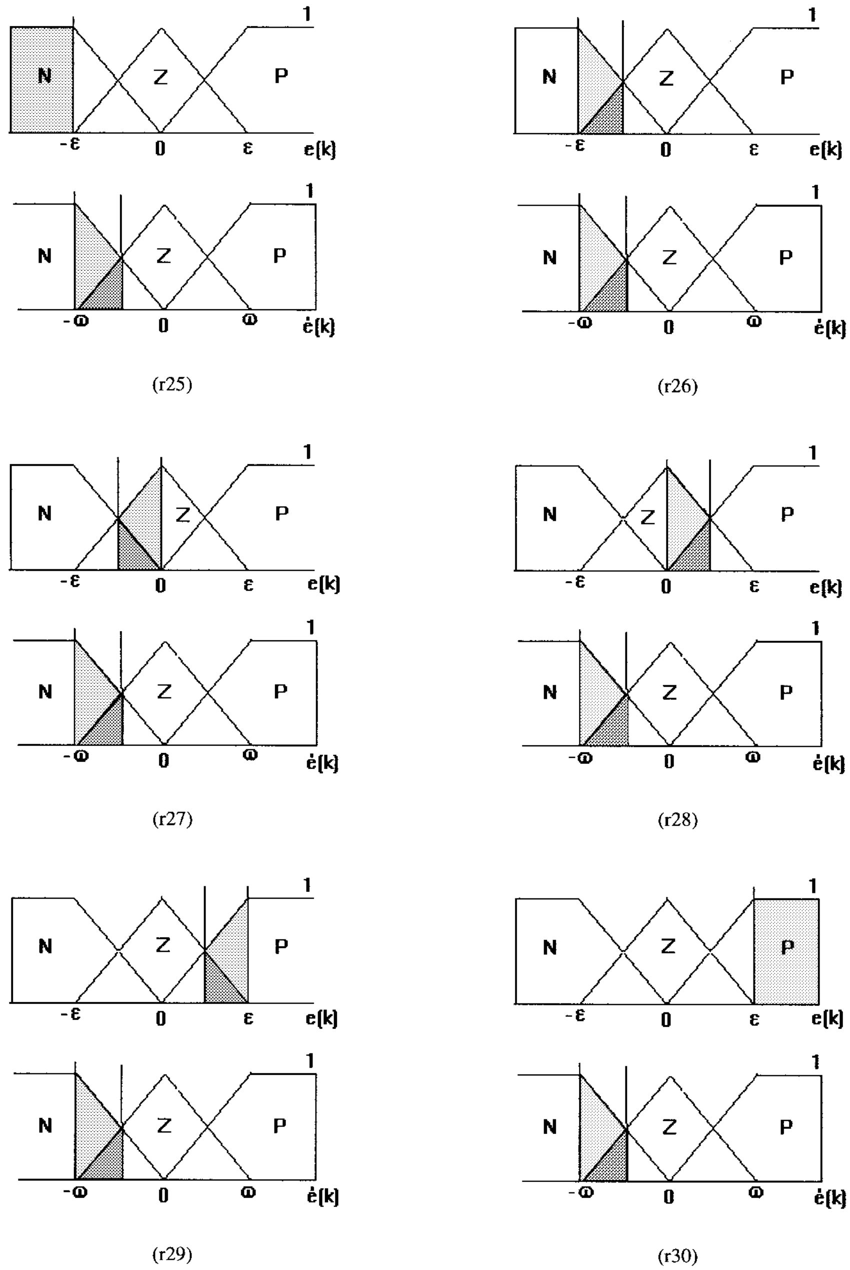
$$\Delta u_f^{(7)}(k) = \frac{(\dot{e}(k) - \omega}{\omega}\epsilon = \lambda \dot{e}(k) - \epsilon \qquad e(k) \leq -\epsilon; \quad 0.5\omega \leq \dot{e}(k) \leq \omega \tag{11}$$

$$\Delta u_f^{(12)}(k) = \epsilon \qquad \epsilon \geq e(k); \quad 0.5\omega \leq \dot{e}(k) \leq \omega \tag{12}$$

$$\Delta u_f^{(13)}(k) = \lambda \dot{e}(k) - \epsilon \qquad e(k) \leq -\epsilon; \quad 0 \leq \dot{e}(k) \leq 0.5\omega \tag{13}$$

$$\Delta u_f^{(18)}(k) = \epsilon \qquad \epsilon \geq e(k); \quad 0 \leq \dot{e}(k) \leq 0.5\omega. \tag{14}$$

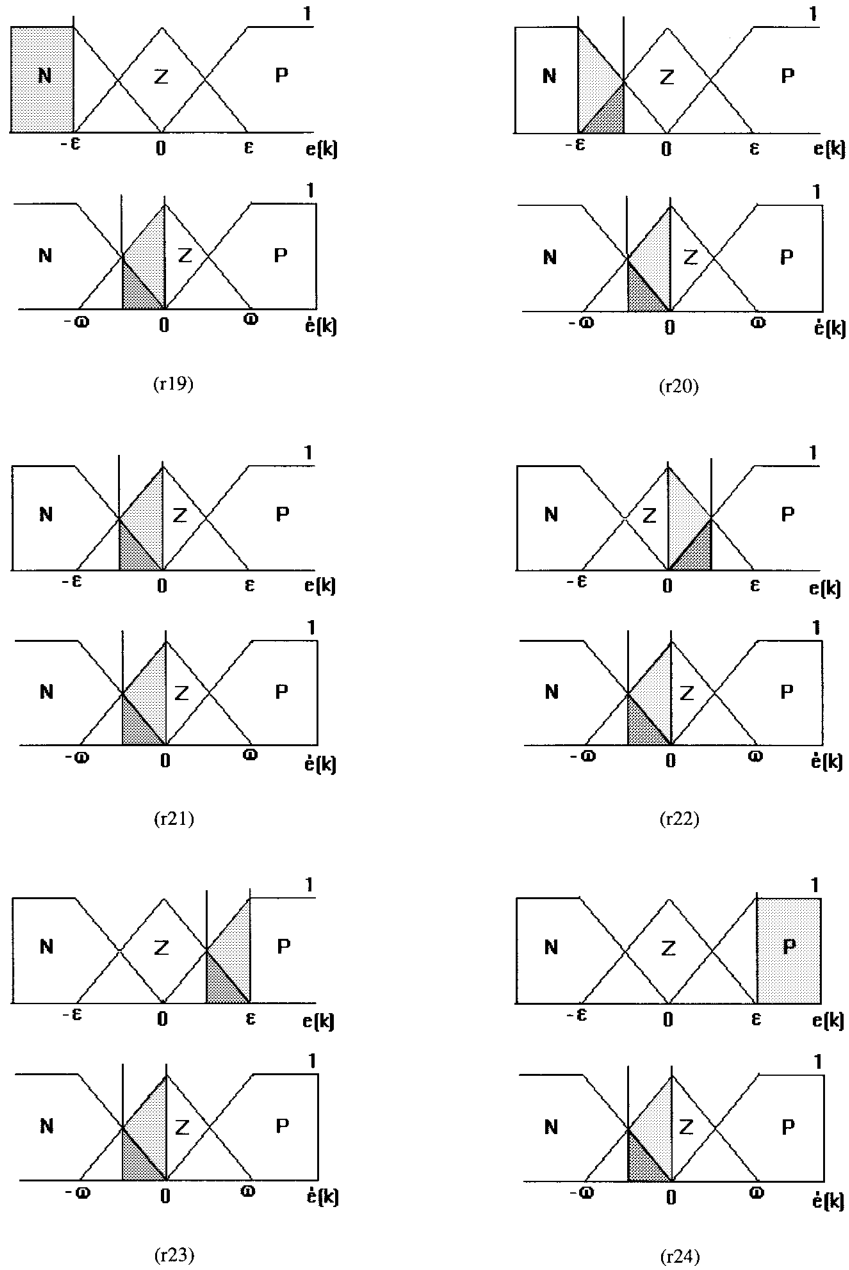
In general, computing $\Delta u_f^{(j)}(k)$ of the third type is rather complex since there are four rules to be activated simultane-



(d)

Fig. 2. *(Continued.)* Formulation of the incremental fuzzy logic controller by fuzzy reasoning.

ously. Here, we describe in detail the computation of $\Delta u_f^{(8)}(k)$ in region 8 $e(k) \in [-\epsilon, -0.5\epsilon]$, $\dot{e}(k) \in [0.5\omega, \omega]$. In this case, each of Rules 1, 2, 4, and 5 contributes the output control actions. For Rule 1, we have the input membership functions values $\mu(e.n) = (e(k)/\epsilon) \geq 0.5$ and $\mu(\dot{e}.p) = (\dot{e}(k)/\omega) \geq 0.5$. Since both of the degrees of membership functions $\mu(e.n)$ and $\mu(\dot{e}.p)$ change in the same range [0.5, 1.0], the MIN-operation is controlled as follows: if $|-e(k)| \leq |\dot{e}(k)|$, $\{\mu(e.n)$ AND $\{\mu(\dot{e}.p)\} = \min\{-(e(k)/\epsilon), (\dot{e}(k)/\omega)\} = -(e(k)/\epsilon)$. If $|-e(k)| > |\dot{e}(k)|$, $\{\mu(e.n)$ AND $\mu(\dot{e}.p)\} = \min\{-(e(k)/\epsilon), (\dot{e}(k)/\omega)\} = (\dot{e}(k)/\omega)$. The corresponding output value is $o.z = 0$. For Rule 2, we have the input membership functions values $\mu(e.z) = (e(k) + \epsilon/\epsilon) \leq 0.5$ and $\mu(\dot{e}.p) = (\dot{e}(k)/\omega) \geq 0.5$. Obviously, $\{\mu(e.z)$ AND $\mu(\dot{e}.p)\} = \min\{(e(k) + \epsilon/\epsilon), (\dot{e}(k)/\omega)\} = (e(k) + \epsilon/\epsilon)$. The corresponding output value is $o.p = \epsilon$. For Rule 4, we have the input membership functions values $\mu(e.n) = (e(k)/\epsilon) \geq 0.5$ and $\mu(\dot{e}.z) = (-\dot{e}(k) + \omega/\omega) \leq 0.5$ and, thus, $\{\mu(e.n)$ AND $\mu(\dot{e}.z)\} = \min\{-(e(k)/\epsilon), (-\dot{e}(k) + \omega/\omega)\} = (-\dot{e}(k) + \omega/\omega)$. The corresponding output value is $o.n = -\epsilon$. For Rule 5, we have the input membership functions' values $\mu(e.z) = (e(k) + \epsilon/\epsilon) \leq 0.5$ and $\mu(\dot{e}.z) = (-\dot{e}(k) + \omega/\omega) \leq 0.5$. In this case, the MIN-operation $\{\mu(e.z)$ AND $\mu(\dot{e}.z)\}$ is controlled as follows: if $|-e(k)| \leq |\dot{e}(k)|$, $\{\mu(e.z)$ AND $\mu(\dot{e}.z)\} = \min\{(e() + \epsilon/\epsilon), (-\dot{e}(k) + \omega/\omega)\} = (-\dot{e}(k) + \omega/\omega)$; if $|[e(k)| > |\dot{e}(k)|$, $\{\mu(e.z)$ AND $\mu(\dot{e}.z)\} = \min\{(e(k) + \epsilon/\epsilon), (-\dot{e}(k) + \omega/\omega)\} = (e(k) + \epsilon/\epsilon)$. The corresponding output value is $o.z = 0$. By the "center of mass" formula (4) we obtain (15), shown at the bottom of page 457. Similarly,



(e)

Fig. 2. *(Continued.)* Formulation of the incremental fuzzy logic controller by fuzzy reasoning.

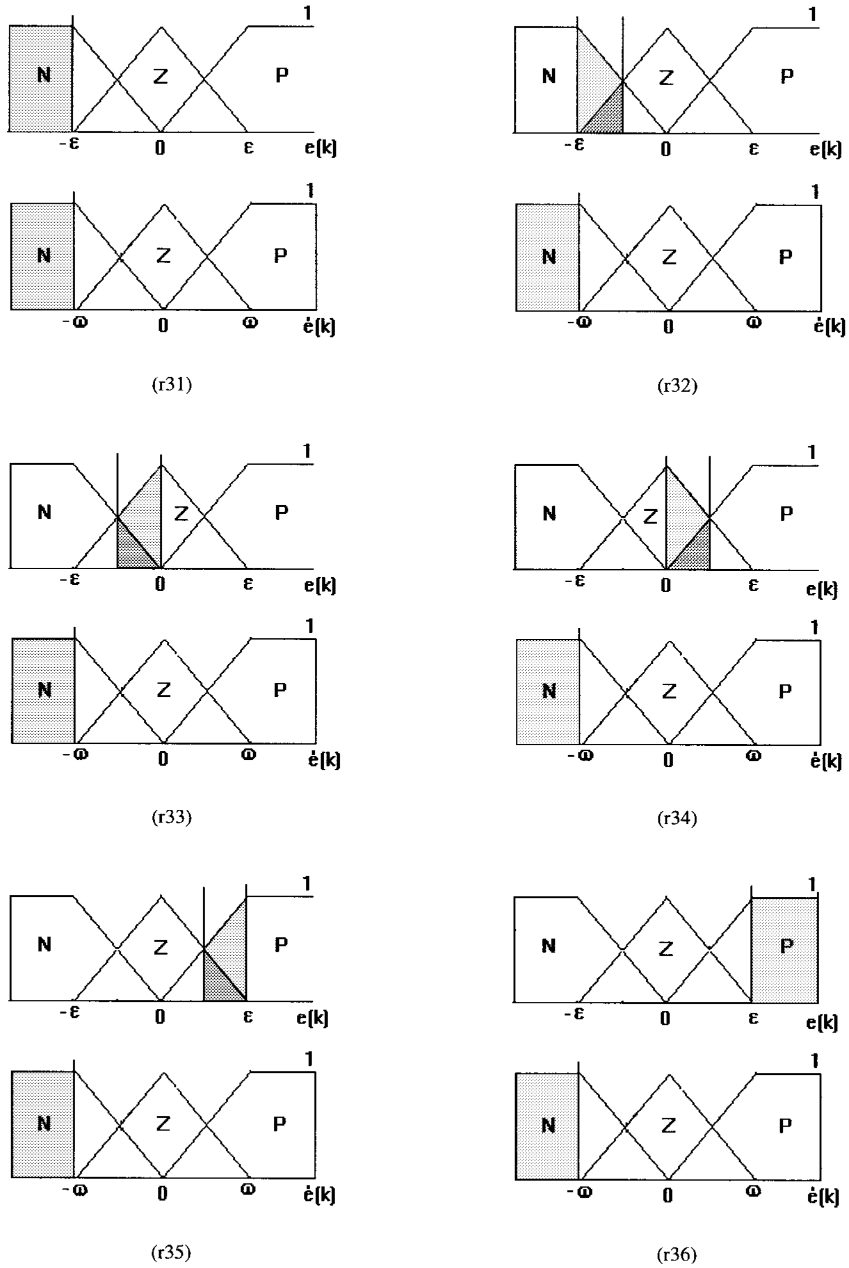
we obtain

$$\Delta u_f^{(9)}(k) = \begin{cases} \dfrac{\epsilon(\lambda\dot{e}(k)+e(k))}{3\epsilon - 2\lambda\dot{e}(k)}, & \text{if } |e(k)+\epsilon| \le |\dot{e}(k)| \\ \dfrac{\epsilon(\lambda\dot{e}(k)+e(k))}{\epsilon - 2e(k)}, & \text{if } |e(k)+\epsilon| > |\dot{e}(k)| \end{cases}$$
$$-0.5\epsilon \le e(k) \le 0; \quad 0.5\omega \le \dot{e}(k) \le \omega \tag{16}$$

$$\Delta u_f^{(10)}(k) = \begin{cases} \dfrac{\epsilon(2\epsilon-\lambda\dot{e}(k))}{3\epsilon - 2\lambda\dot{e}(k)}, & \text{if } |-e(k)+\epsilon| \le |\dot{e}(k)| \\ \dfrac{\epsilon(\lambda\dot{e}(k)+2e(k))}{\epsilon + 2e(k)}, & \text{if } |-e(k)+\epsilon| > |\dot{e}(k)| \end{cases}$$
$$0 \le e(k) \le 0.5\epsilon; \quad 0.5\omega \le \dot{e}(k) \le \omega \tag{17}$$

$$\Delta u_f^{(11)}(k) = \begin{cases} \dfrac{\epsilon(2\epsilon-\lambda\dot{e}(k))}{3\epsilon - 2\lambda\dot{e}(k)}, & \text{if } |-e(k)| \le |\dot{e}(k)| \\ \dfrac{\epsilon(2\epsilon-e(k))}{3\epsilon - 2e(k)}, & \text{if } |e(k)| > |\dot{e}(k)| \end{cases}$$
$$0.5\epsilon \le e(k) \le \epsilon; \quad 0.5\omega \le \dot{e}(k) \le \omega \tag{18}$$

$$\Delta u_f^{(14)}(k) = \begin{cases} \dfrac{\epsilon(e(k)+\lambda\dot{e}(k))}{3\epsilon + 2e(k)}, & \text{if } |-e(k)|+\epsilon| \le |\dot{e}(k)| \\ \dfrac{\epsilon(\lambda\dot{e}(k)+e(k))}{\epsilon + 2\lambda\dot{e}(k)}, & \text{if } |-e(k)+\epsilon| > |\dot{e}(k)| \end{cases}$$
$$-\epsilon \le e(k) \le -0.5\epsilon; \quad 0 \le \dot{e}(k) \le 0.5\omega \tag{19}$$



(f)

Fig. 2. *(Continued.)* Formulation of the incremental fuzzy logic controller by fuzzy reasoning.

$$\Delta u_f^{(15)}(k) = \begin{cases} \dfrac{\epsilon(\lambda \dot{e}(k) + e(k))}{\epsilon - 2e(k)}, & \text{if } |e(k)| \le |\dot{e}(k)| \\ \dfrac{\epsilon(\lambda \dot{e}(k) + e(k))}{\epsilon + 2\lambda \dot{e}(k)}, & \text{if } |e(k)| > |\dot{e}(k)| \end{cases}$$
$$-0.5\epsilon \le e(k) \le 0; \quad 0 \le \dot{e}(k) \le 0.5\omega \tag{20}$$

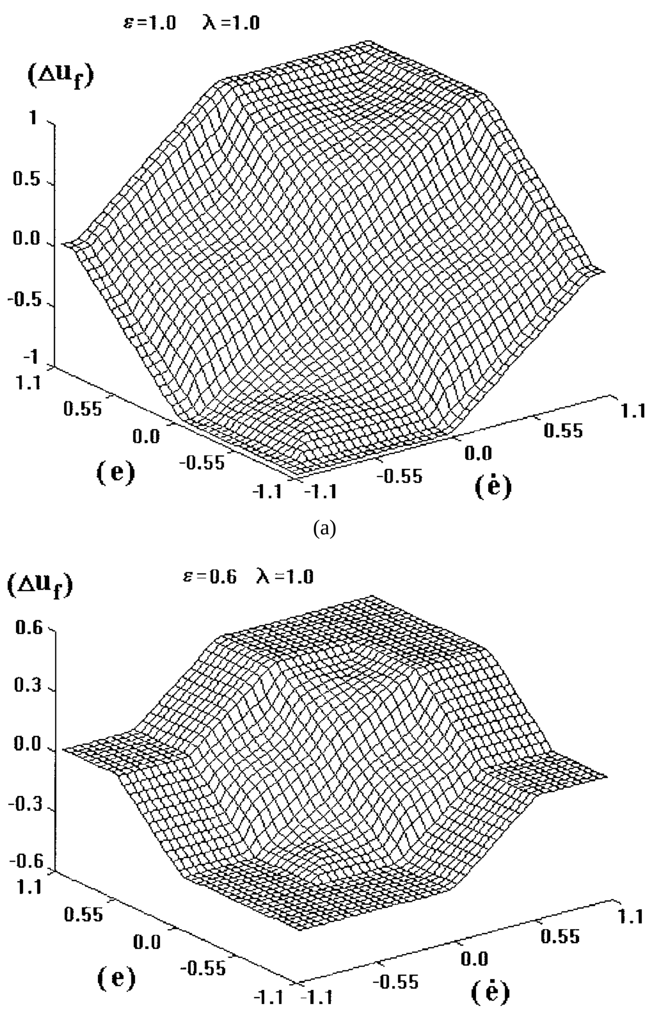
$$\Delta u_f^{(16)}(k) = \begin{cases} \dfrac{\epsilon(\lambda \dot{e}(k) + 2e(k))}{\epsilon + 2e(k)}, & \text{if } |e(k)| \le |\dot{e}(k)| \\ \dfrac{\epsilon(2\lambda \dot{e}(k) + e(k))}{\epsilon + 2\lambda \dot{e}(k)}, & \text{if } |e(k)| \le |\dot{e}(k)| \end{cases}$$
$$0 \le e(k) \le 0.5\epsilon; \quad 0 \le \dot{e}(k) \le 0.5\omega \tag{21}$$

$$\Delta u_f^{(17)}(k) = \begin{cases} \dfrac{\epsilon(2\epsilon - e(k))}{3\epsilon - 2e(k)}, & \text{if } |-e(k) + \epsilon| \le |\dot{e}(k)| \\ \dfrac{\epsilon(2\lambda \dot{e}(k) + e(k))}{\epsilon + 2\lambda \dot{e}(k)}, & \text{if } |-e(k) + \epsilon| > |\dot{e}(k)| \end{cases}$$
$$0.5\epsilon \le e(k) \le \epsilon; \quad 0 \le \dot{e}(k) \le 0.5\omega. \tag{22}$$

On the basis of (6), we construct $\Delta u_f(k) = FLC(e(k)\ \dot{e}(k)) = \{\Delta u_f^{(i)}(k)\}$ $(i = 1, \cdots, 36)$ by using (7)–(22). It should be noted that in $\Delta u_f(k)$ there exists a parameter ϵ. In a later discussion, we can see that this parameter is only one to be tuned for achieving a desired control performance. Fig. 3(a) and (b) shows the control surfaces of the incremental fuzzy logic controller with $(\epsilon = 1.0;\ \lambda = 1)$ and $(\epsilon = 0.6;\ \lambda = 1)$, respectively.

TABLE I
RULE BASE OF THE INCREMENTAL FUZZY LOGIC CONTROLLER

$\dot{e}(k)$ \ $\Delta u_f(k)$ \ $e(k)$	N	Z	P
N	N	N	Z
Z	N	Z	P
P	Z	P	P



(a)

(b)

Fig. 3. Control surfaces of the incremental fuzzy logic controller.

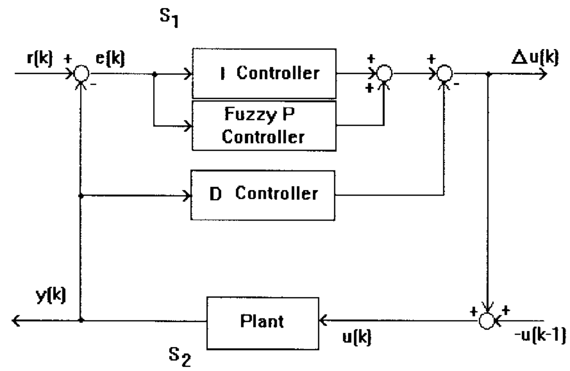


Fig. 4. A nonlinear control system.

IV. STABILITY CONDITIONS

The "small gain theorem" gives sufficient conditions under which a "bounded input" yields a "bounded output" [19], [20]. In [10] and [11], this theorem is used to analyze the bounded-input/bounded-output stability of the fuzzy PD and fuzzy PI+fuzzy D control systems, respectively. In this section, we use this methodology to derive the stability conditions for the proposed fuzzy P+ID controller. A control structure of the propose fuzzy P+ID is shown in Fig. 4. Being similar to the result in [11], the sufficient condition for the bounded-input/bounded output stability of the nonlinear control system is given as follows:

$$\begin{cases} \left\| S_1\left(\begin{bmatrix} e(k) \\ y(k) \end{bmatrix}\right)\right\| \le M_1 + L_1 \left\| \begin{bmatrix} e(k) \\ y(k) \end{bmatrix} \right\| \\ \|S_2(u(k))\| \le M_2 + L_2\|u(k)\| \end{cases} \tag{23}$$

for all $t \in [0, \infty)$, where M_1, M_2, L_1, and L_2 are constants with L_1, $L_2 \ge 0$, and $L_1 L2 < 1$.

For a conventional PID controller in (1), we have

$$\begin{aligned} \left\| S_1\left(\begin{bmatrix} e(k) \\ y(k) \end{bmatrix}\right)\right\| &= \left\| \begin{bmatrix} K_P + K_I T & 0 \\ 0 & \dfrac{K_D}{T} \end{bmatrix} \begin{bmatrix} e(k) \\ y(k) \end{bmatrix} \right\| \\ &\le \left\| \begin{bmatrix} K_P + K_I T & 0 \\ 0 & \dfrac{K_D}{T} \end{bmatrix} \right\| \left\| \begin{bmatrix} e(k) \\ y(k) \end{bmatrix} \right\| \\ &\le \max\left\{ \|K_P + K_I T\|, \left\| \frac{K_D}{T} \right\| \right\} \\ &\quad \cdot \max\{\|e(k)\|, \|y(k)\|\} \end{aligned} \tag{24}$$

$$\|S_2(e_2(k))\| = \|\mathrm{N}(e_2(k))\| = \|\mathrm{N}\|\,|e_2(k)| \tag{25}$$

where $\|\mathrm{N}\|$ is the operator norm of the given $\mathrm{N}(\cdot)$ or the gain of the given nonlinear system defined as usual by

$$\|\mathrm{N}\| := \sup_{v_1 \ne v_2,\, k \ge 1} \frac{|\mathrm{N}(v_1(k)) - \mathrm{N}(v_2(k))|}{|v_1(k) - v_2(k)|}. \tag{26}$$

$$\Delta u_f^{(8)}(k) = \begin{cases} \dfrac{\mu(e.n) \times o.z + \mu(e.z) \times o.p + \mu(\dot{e}.z) \times o.n + \mu(\dot{e}.z) \times o.z}{\mu(e.n) + \mu(e.z) + \mu(\dot{e}.z) + \mu(\dot{e}.z)} = \dfrac{\epsilon(\lambda \dot{e}(k) + e(k))}{3\epsilon - 2\lambda \dot{e}(k)} & \text{if } |-e(k)| \le |\dot{e}(k)| \\ \dfrac{\mu(\dot{e}.p) \times o.z + \mu(e.z) \times o.p + \mu(\dot{e}.z) \times o.n + \mu(e.z) \times o.z}{\mu(\dot{e}.p) + \mu(e.z) + \mu(\dot{e}.z) + \mu(e.z)} = \dfrac{\epsilon(\lambda \dot{e}(k) + e(k))}{3\epsilon + 2e(k)} & \text{if } |-e(k)| > |\dot{e}(k)| \end{cases}$$

$$-\epsilon \le e(k) \le -0.5\epsilon; \quad 0.5\omega \le \dot{e}(k) \le \omega \tag{15}$$

In comparison of (24) and (25) with (23), it is easy to obtain the constants L_1 and L_2 for the PID controller

$$\begin{cases} L_1 = \max\left\{|K_{\rm P}+K_{\rm I}T|, \left|\dfrac{K_{\rm D}}{T}\right|\right\} \\ L_2 = \|{\rm N}\|. \end{cases} \tag{27}$$

If the following condition

$$|K_{\rm D}| \ge |TK_{\rm P}+K_{\rm I}T^2| \tag{28}$$

is satisfied, the sufficient condition for the nonlinear system by the PID controller is expressed by

$$L_1L_2 = \left|\frac{K_{\rm D}}{T}\right| \|{\rm N}\| < 1. \tag{29}$$

The difference between the proposed fuzzy P+ID controller and the PID controller is their proportional terms. This terms of the incremental fuzzy P+ID controller can be expressed by

$$\begin{aligned} K_{\rm P}^*\Delta u_f(k) &= K_{\rm P}^* FLC(e(k),\dot e(k)) = K_{\rm P}^* FLC(e(k), f(e(k))) \\ &= K_{\rm P}^*\Phi(e(k)) \end{aligned} \tag{30}$$

where $\|\Phi\|$ is the gain of the given incremental fuzzy controller and will be derived late. Thus, for the fuzzy P+ID controller we have

$$\begin{aligned} \left\| S_1\left(\begin{bmatrix} e(k)\\ y(k)\end{bmatrix}\right)\right\| &= \left\|\begin{bmatrix} \|\Phi\|K_{\rm P}^*+K_{\rm I}T & 0 \\ 0 & \dfrac{K_{\rm D}}{T}\end{bmatrix}\begin{bmatrix} e(k)\\ y(k)\end{bmatrix}\right\| \\ &\le \left\|\begin{bmatrix} \|\Phi\|K_{\rm P}^*+K_{\rm I}T & 0 \\ 0 & \dfrac{K_{\rm D}}{T}\end{bmatrix}\right\| \left\|\begin{bmatrix} e(k)\\ y(k)\end{bmatrix}\right\| \\ &\le \left\{ \|\,\|\Phi\|K_{\rm P}^*+K_{\rm I}T\|, \left\|\frac{K_{\rm D}}{T}\right\|\right\} \\ &\quad\cdot \max\{\|e(k)\|, \|y(k)\|\}. \end{aligned} \tag{31}$$

Then, we obtain

$$\begin{cases} L_1 = \max\left\{|\,\|\Phi\|K_{\rm P}^*+K_{\rm I}T|, \left|\dfrac{K_{\rm D}}{T}\right|\right\} \\ L_2 = \|{\rm N}\|. \end{cases} \tag{32}$$

If the following additional condition

$$|K_{\rm D}| \ge \left|\,\|\Phi|TK_{\rm P}^*+K_{\rm I}T^2\right| \tag{33}$$

is satisfied, one can replace the conventional PID controller by the fuzzy P+ID controller while preserving the same stability.

Now we use $\Delta u_f(k) = FLC(e(k),\dot e(k))$ derived in Section III to compute $\|\Phi\|$. For $\Delta u_f^{(i)}(k)$ of the first and second types, it is easy to obtain

$$\|\Phi\| = \|\Phi^{(1,4,5,6,12,18)}\| = 0 \tag{34}$$

$$\|\Phi\| = \|\Phi^{(2,3)}\| = 1 \tag{35}$$

$$\|\Phi\| = \|\Phi^{(7,13}\| = \frac{\lambda}{T}. \tag{36}$$

For $\Delta u_f^{(8)}(k)$, we have

$$\begin{aligned} &\Delta u_f^{(8)}(k) \\ &= \begin{cases} \dfrac{\epsilon(\lambda+T)e(k)-\epsilon\lambda Te(k-1)}{T(3\epsilon-2\dot e(k))}, & \text{if } |-e(k)| \le |\dot e(k)| \\ \dfrac{\epsilon(\lambda+T)e(k)-\epsilon\lambda e(k-1)}{T(3\epsilon+2e(k))}, & \text{if } |-e(k)| > |\dot e(k)| \end{cases} \\ &\quad -\epsilon \le e(k) \le -0.5\epsilon; \quad 0.5\omega \le \dot e(k) \le \omega. \end{aligned} \tag{37}$$

For $|-e(k)| \le \dot e(k)|$, let $\dot e(k) = n\omega$ while for $|-e(k)| > |\dot e(k)|$, let $e(k) = -\epsilon$. Then we obtain

$$\Delta u_f^{(8)}(k) \le \frac{(\lambda+T)e(k)-\lambda e(k-1)}{T} \tag{38}$$

$$\|\Phi\| = \|\Phi^{(8)}\| \le \frac{\lambda+T}{T}. \tag{39}$$

In the same manner, we can prove the operator norm $\|\Phi\|$ in regions 9, 14, 15 as follows:

$$\|\Phi^{(9,14,15)}\| = \|\Phi^{(8)}\| \le \frac{\lambda+T}{T} \tag{40}$$

For $\Delta u_f^{(10)}(k)$, we have (41), shown at the bottom of the next page. For $|-e(k)+\epsilon| \le |\dot e(k)|$, let $\dot e(k) = \omega$ while for $|-e(k)+\epsilon| > |\dot e(k)|$, let $e(k) = 0$. Then, we obtain

$$\begin{aligned} &\Delta u_f^{(10)}(k) \\ &\le \begin{cases} \dfrac{-\lambda e(k)+\lambda e(k-1)+2\epsilon T}{T}, & \text{if } |-e(k)+\epsilon| \le |\dot e(k)| \\ \dfrac{(\lambda+2T)e(k)-\lambda e(k-1)}{T}, & \text{if } |-e(k)+\epsilon| > |\dot e(k)| \end{cases} \end{aligned} \tag{42}$$

$$\|\Phi\| = \|\Phi^{(10)}\| \le \begin{cases} \dfrac{\lambda}{T}, & \text{if } |-e(k)+\epsilon| \le |\dot e(k)| \\ \dfrac{\lambda+2T}{T}, & \text{if } |-e(k)+\epsilon| > |\dot e(k)|. \end{cases} \tag{43}$$

For $\Delta u_f^{(11)}(k)$, we have

$$\begin{aligned} &\Delta u_f^{(11)}(k) \\ &= \begin{cases} \dfrac{-\epsilon\lambda e(k)+\epsilon\lambda e(k-1)+2\epsilon^2}{T(3\epsilon-2\lambda\dot e(k))}, & \text{if } |e(k)| \le |\dot e(k)| \\ \dfrac{-\epsilon e(k)-2\epsilon^2}{3\epsilon-2e(k)}, & \text{if } |e(k)| > |\dot e(k)| \end{cases} \\ &\quad 0.5\epsilon \le e(k) \le \epsilon; \quad 0.5\omega \le \dot e(k) \le \omega. \end{aligned} \tag{44}$$

For $|e(k)| \le |\dot e(k)|$, let $\dot e(k) = \epsilon$ while for $|e(k)| > |\dot e(k)|$, $e(k) = \epsilon$. Then we have

$$\Delta u_f^{(11)}(k) \le \begin{cases} \dfrac{-\lambda e(k)+2\epsilon+\lambda e(k-1)}{T}, & \text{if } |e(k)| \le |\dot e(k)| \\ -e(k)-2\epsilon, & \text{if } |e(k)| > |\dot e(k)| \end{cases} \tag{45}$$

$$\|\Phi\| = \|\Phi^{(11)}\| \le \begin{cases} \dfrac{\lambda}{T}, & \text{if } |e(k)| \le |\dot e(k)| \\ 1, & \text{if } |e(k)| < |\dot e(k)|. \end{cases} \tag{46}$$

For $\Delta u_f^{(16)}$ we have

$$\begin{aligned} &\Delta u_f^{(16)}(k) \\ &= \begin{cases} \dfrac{\epsilon(\lambda+2T)e(k)-\epsilon\lambda e(k-1)}{T(\epsilon+2e(k))}, & \text{if } |e(k)| \le |\dot e(k)| \\ \dfrac{\epsilon(2\lambda+T)e(k)-2\epsilon\lambda e(k-1)}{T(\epsilon+2\lambda\dot e(k))}, & \text{if } |e(k)| > |\dot e(k)| \end{cases} \\ &\quad 0 \le e(k) \le 0.5\epsilon; \quad 0 \le \dot e(k) \le 0.5\omega. \end{aligned} \tag{47}$$

For $|e(k)| \le |\dot e(k)|$, let $\dot e(k) = 0$ while for $|e(k)| > |\dot e(k)|$,

$e(k) = 0$. Then we have

$$\Delta u_f^{(16)}(k) \le \begin{cases} \dfrac{(\lambda+2T)e(k) - e(k-1)}{T}, & \text{if } |e(k)| \le |\dot{e}(k)| \\ \dfrac{(2\lambda+T)e(k) - 2e(k-1)}{T}, & \text{if } |e(k)| > |\dot{e}(k)| \end{cases}$$
$$0 \le e() \le 0.5\epsilon; \quad 0 \le \dot{e}(k) \le 0.5\omega \tag{48}$$

$$\|\Phi\| = \|\Phi^{(16)}\| \le \begin{cases} \dfrac{(\lambda+2T)}{T}, & \text{if } |e(k)| \le |\dot{e}(k)| \\ \dfrac{2\lambda+T}{T}, & \text{if } |e(k)| > |\dot{e}(k)|. \end{cases} \tag{49}$$

For $\Delta u_f^{(17)}(k)$ in (22) we have

$$\Delta u_f^{(17)}(k) = \begin{cases} \dfrac{-\epsilon e(k)+2\epsilon^2}{3\epsilon-2e(k)}, & \text{if } |-e(k)+\epsilon| \le |\dot{e}(k)| \\ \dfrac{\epsilon(2\lambda+T)e(k)-2\epsilon\lambda e(k-1)}{T(\epsilon+2\lambda\dot{e}(k))}, & \text{if } |-e(k)+\epsilon| > |\dot{e}(k)| \end{cases}$$
$$0.5\epsilon \le (k) \le \epsilon; \quad 0 \le \dot{e}(k) \le 0.5\omega. \tag{50}$$

For $|-e(k)+\epsilon| \le |\dot{e}(k)|$, let $e(k) = \epsilon$ while for $|e(k)| > |\dot{e}(k)|$, $\dot{e}(k) = 0$. Then we have

$$\Delta u_f^{(17)}(k) \le \begin{cases} -e(k)+2\epsilon, & \text{if } |-e(k)+\epsilon| \le |\dot{e}(k)| \\ \dfrac{(2\lambda+T)e(k)-2\lambda e(k-1)}{T}, & \text{if } |-e(k)+\epsilon| > |\dot{e}(k)| \end{cases} \tag{51}$$

$$\|\Phi\| = \|\Phi^{(17)}\| \le \begin{cases} 1, & \text{if } |-e(k)+\epsilon| \le |\dot{e}(k)| \\ \dfrac{2\lambda+T}{T}, & \text{if } |-e(k)+\epsilon| > |\dot{e}(k)|. \end{cases} \tag{52}$$

Summarizing, we have

$$\|\Phi\| \le \left\{0, 1, \frac{\lambda}{T}, \frac{\lambda+T}{T}, \frac{\lambda+2T}{T}, \frac{2\lambda+T}{T}\right\} \tag{53}$$

where T is the sampling period. Since the sample period T is smaller than one and we let $\lambda = 1$, we have $\max\|\Phi\| = (2 + T/T)$. In our study, we also choose $K_P^* = K_P$, hence, the condition expressed by (34) becomes

$$|K_D| \ge |(T+2)K_P + K1T^2|. \tag{54}$$

This result shows that this condition for the fuzzy P+ID is stronger than that of the original PID-type controller in (28). Besides, the stability of the system is independent of the fuzzy controller parameter e.

V. Approaches to Design of the Controller

The works in [22]–[25] proposed different methods for tuning of the PID control parameters. Because these methods do not need to use any model of a controlled object, they are still widely used in industrial applications. In order to take this advantage, we systematically present the design process based on the Ziegler and Nichols' approach. In the Ziegler and Nichols technique, the parameter tuning is based on the stability limits of a system. The derivative and integral terms are initially put out of the system and proportional gain is increased until the critical oscillation point (critical gain $K_{P(CRIT)}$ and period $T_{(CRIT)}$. Then the PID controller parameters are selected as follows:

$$K_P = 0.6K_{P(CRIT)}$$
$$K_I = \frac{2.0K_P}{T_{(CRIT)}}$$
$$K_D = 0.12K_P T_{(CRIT)}. \tag{55}$$

The purpose for design of the fuzzy P+ID controller is to improve the control performance of the industrial plants without deteriorating the stability. Since the fuzzy P+ID controller is constructed by replacing the conventional proportional term with the fuzzy one, we propose the following formula:

$$K_P = 0.6K_{P(CRIT)}$$
$$K_I = \frac{2.0K_P}{T_{(CRIT)}}$$
$$K_D = (T+2)K_P + K_I T^2 \tag{56}$$

for determination of their parameters. We select the parameter K_D of the derivative controller by using the sufficient stability condition in (54) instead of the Ziegler and Nichols' formula. This result implies that stability of a system does not change after the conventional PID controller is replaced by the fuzzy P+ID controller without modifying any PID-type controller parameter. The selection of the sampling period T is done in two stages: 1) during the loop design and 2) during the controller design. The empirical rule of Franklin and Powell [21] suggests that the sampling frequency must be from 4 to 20 times the bandwidth of the closed-loop system. For the controller design, T should be increased to be greater than the sum of the error computation time, the digital analogue converter (DAC) and analogue digital converter (ADC) conversion times, and the zero-order hold delay time. In selecting T, one must have in mind that

1) if T is greater, the stability regions are smaller;
2) large T implies small cost;
3) large T results in large conversion times of the DAC's and ADC's (i.e., to smaller cost);

$$\Delta u_f^{(10)}(k) = \begin{cases} \dfrac{-\epsilon\lambda e(k) + \epsilon\lambda e(k-1) + 2\epsilon^2}{T(3\epsilon - 2\lambda\dot{e}(k))}, & \text{if } |-e(k)+\epsilon| \le |\dot{e}(k)| \\ \dfrac{\epsilon e(k)\,(\lambda+2T) - \epsilon\lambda e(k-1)}{T(\epsilon+2e(k))}, & \text{if } |-e(k)+\epsilon| > |\dot{e}(k)| \end{cases}$$
$$0 \le e(k) \le 0.5\epsilon; \quad 0.5\omega \le \dot{e}(k) \le \omega. \tag{41}$$

4) small T allows good system performance in the presence of noise.

In a steady-state stage, we have $\dot{e}(k) \approx 0.0$ and $|e(k)| > |\dot{e}(k)|$. For PID control, it is ease to obtain $\Delta u(k) \approx K_{\rm I}Te(k)$ while for fuzzy P+ID control, $\Delta u_f^{(15,16)}(k)$ or $\Delta u_f^{(21,22)}(k)$ are activated. Because of symmetry, we only analyze the case of $\Delta u_f^{(15,16)}(k)$. Obviously, $\Delta u_f^{(15,16)}(k) \approx e(k)$, hence, it is ease to obtain $\Delta u(k) \approx K_{\rm P}e(k) + K_{\rm I}Te(k) = (K_{\rm P} + K_{\rm I}T)e(k)$. This implies that the fuzzy proportional term becomes an additional integral term $K_{\rm P}e(k)$. Because $(K_{\rm P} + K_{\rm I}T) > K_{\rm I}T$, the fuzzy P+ID controller reduces greatly the steady-state error. In this case, we have also $e(k)_\infty \leq 0.5\epsilon$. Thus, a small steady-state error can be achieved by selecting a small ϵ. In a starting process, the fuzzy P controller outputs a constant control signal $K_{\rm P}\epsilon$. If the control parameter e is too small, a rise time response slows down. Here,$0.1 < \epsilon < 2.0$ is recommended.

When the controlled object is described by any modeling method, such as behavior modeling [17], the fuzzy P+ID controller can be automatically tuned by using genetic algorithms. In order to optimize the fuzzy P+ID controller, the integral-of-time-multiplied absolute-error (ITAE) criterion

$$\Omega = \int_0^{t_0} t|e(t)|\, dt \tag{57}$$

is used to describe control performance. We briefly introduce the design idea as follows. Since the fuzzy P+ID parameters $K_{\rm P}$, $K_{\rm I}$, $K_{\rm D}$, and ϵ influence Ω, the optimization of the fuzzy P+ID controller is the computation of the minimum value Ω^* by searching for the corresponding parameters $K_{\rm P}$, $K_{\rm I}$, $K_{\rm D}$, and ϵ. Here, a genetic algorithm (GA) is used to optimize these parameters [26]. At first, we map the parameters $K_{\rm P}$, $K_{\rm I}$, and ϵ into binary strings in two steps. The binary strings used in this paper are five digital strings. One of the major operators in GA is crossover. In a population of size n, $n/2$ individuals (parents) are selected to take part in crossover for every generation. Here, the sufficient condition (55) is used as an additional restrict to compute a fitness. All selected parents have better fitness value than other else in the population. Every pair of parents are selected randomly without repeat in the crossover population and produce two offsprings simultaneously after crossover. For crossover individual, we use single-point crossover on all strings of the individual. So all four strings of the individual do crossover simultaneously with related ones each other at four sites given randomly. The four crossover sites may be the same or different from these four strings. Another of the major operators in SGA is mutation. Mutation takes place if the fitness value of some individuals is considered to be "bad" or any constraint is violated. Mutation takes place in two different ways. In first several generations, all strings of an individual take part in mutation. The purpose is that the global optimal region could not be neglected. In all later generations, only two strings of an individual take part in mutation. After crossover and mutation processes, a new population is generated. The population size may be greater than previous one. Some individuals are cut with "bad" fitness value from this population so in a new generation the population size remains unchanged. This search process stops until the minimum value Ω^*, that is the optimal parameters $K_{\rm P}$, $K_{\rm I}$, $K_{\rm D}$, and ϵ, of the fuzzy P+ID controller are obtained.

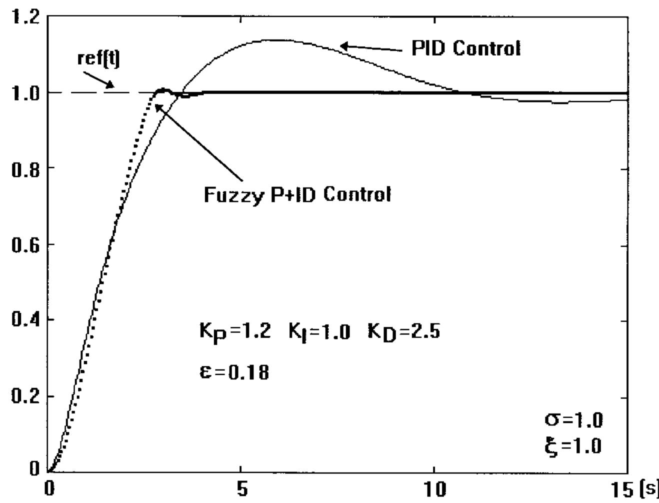


Fig. 5. Time responses of PID and fuzzy P+ID control.

VI. Numerical Simulations

In order to demonstrate the effectiveness and robustness of the proposed fuzzy P+ID controller, by numerical simulations we test time responses for the control of the following nonlinear plant

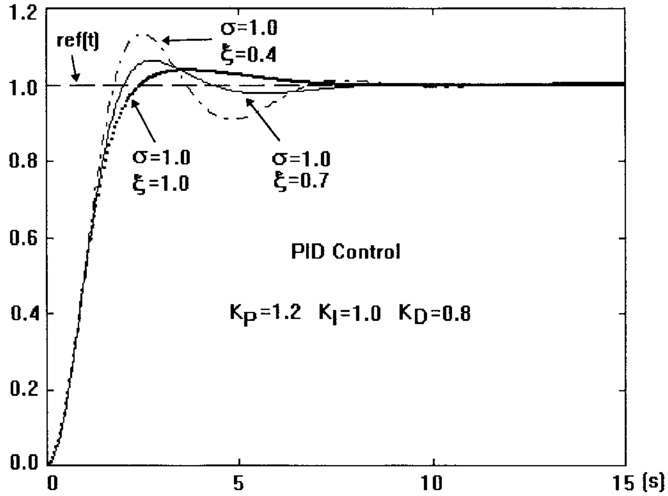
$$\ddot{y} + 2.0\xi\sigma\dot{y}y + \sigma^2 y^2 = \sigma^2 u. \tag{58}$$

In all simulation studies, let the sampling period $T = 0.05$s.

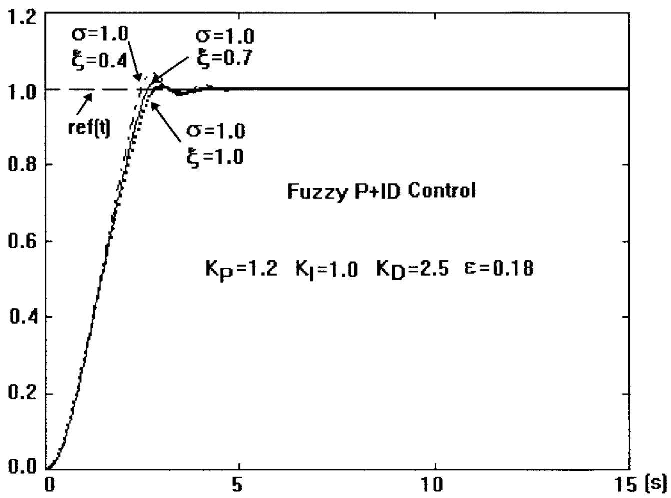
A. Comparison of Time Responses Under Step Control

Here we choose $K_{\rm P} = 1.2$ and $K_{\rm I} = 1.0$ and obtain $K_{\rm D} = 2.5 > (T+2)K_{\rm P} + T^2K_{\rm I} = 2.4625$ according to the sufficient stability condition (54). Now we use the designed PID-type controller to control the nonlinear plant. In this simulation, the nonlinear plant has the parameters $\sigma = 1.0$ and $\xi = 1.0$ and the initial values of the plant $y(0)$, $y'(0)$, and $y''(0)$ are zero. The thin-solid curve in Fig. 5 represents the step response of the nonlinear system yielded by the PID controller. Then we use the fuzzy P+ID controller with $\epsilon = 0.18$ to replace the PID controller without modifying any original parameters. The thick-dot curve in Fig. 5 represents the step response yielded by the fuzzy P+ID controller. It is very clear that the control performance is improved.

In order to compare the robustness of the step control while changing parameters of the plants, we fine the PID-type controller by selecting the controller's parameter as $K_{\rm D} = 0.8$ with regard to the plant's parameters $\sigma = 1.0$ and $\xi = 1.0$. Using this PID-type controller, a quite good control performance is also achieved, as shown in Fig. 6(a). Then, the plant's parameters are redefined as ($\sigma = 1.0$, $\xi = 0.7$) and ($\sigma = 1.0$, $\xi = 0.4$). Their dynamic behaviors are represented by the thin-solid and thin-dashed curves in



(a)



(b)

Fig. 6. Step time responses of step control while changing plant's parameters. (a) PID control. (b) Fuzzy P+ID control.

Fig. 6(a), respectively. Meanwhile, Fig. 6(b) shows the step responses of the plant controlled by the fuzzy P+ID controller. It is clear that the dynamic behaviors, yielded by the fuzzy P+ID controller, exhibit much better control performance than those by the PID-type controller. Now we add a disturbance $y_{\rm dis} = 0.3\sin(1.1t)$ to the plant with $\sigma = 1.0$ and $\xi = 1.0$. Fig. 7 shows dynamic behaviors of PID-type control and of the fuzzy P+ID control, as shown by thin-solid and thick-dot curves, respectively. In spite of the disturbance, the control performance yielded by the fuzzy P+ID controller is much better than that yielded by the PID-type controller.

B. Comparison of Time Responses Under Tracking Control

Now the reference signal is chosen as a ramp function $\mathrm{ref}(t) = 0.1t$ and the initial values of the plant are chosen as $y(0) = 0.1$, $y'(0) = 0$, and $y''(0) = 0$. The plant with $\sigma = 1.0$ and $\xi = 1.0$ is controlled by the PID-type controller with ($K_{\rm P} = 1.2$, $K_{\rm I} = 1.0$, $K_{\rm D} = 0.8$), which is tuned for the step control in Fig. 6(a). Its dynamic behavior represented by the thin-dashed curve in Fig. 8 shows a quite large error during tracking process. In order to reduce this error, the PID-type controller's parameters are refined by selecting ($K_{\rm P} = 7.2$, $K_{\rm I} = 3.5$, $K_{\rm D} = 3.8$). By using the fuzzy P+ID controller with the same parameters above, however, the response behavior exhibits a very small tracking error after starting stage.

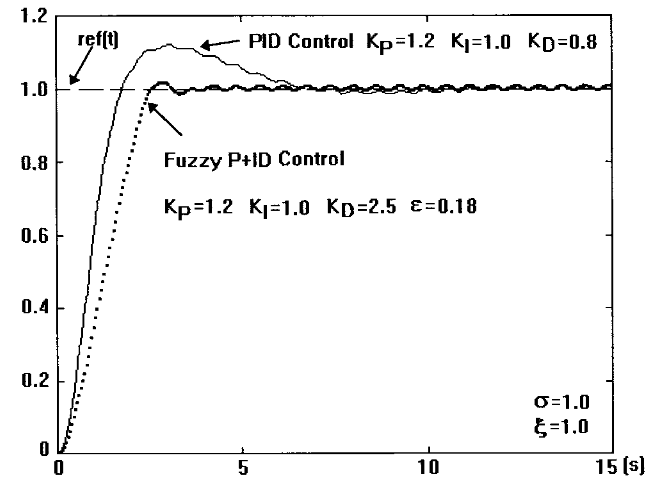


Fig. 7. Time responses of step control with a disturbance.

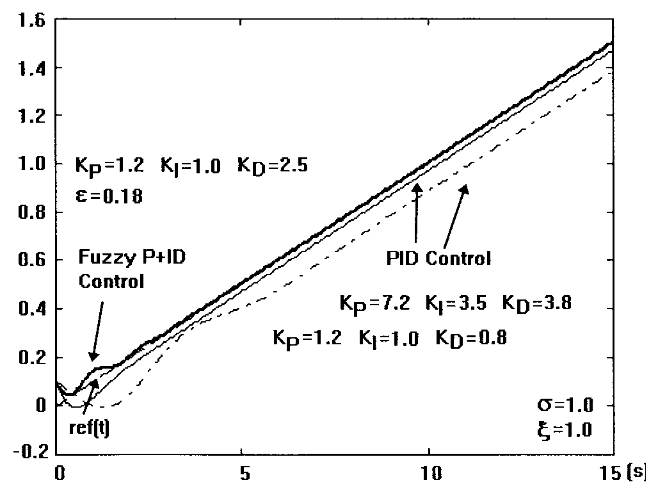
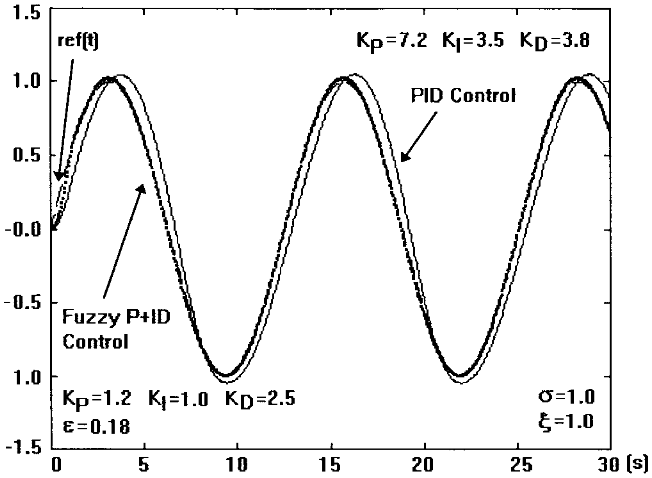


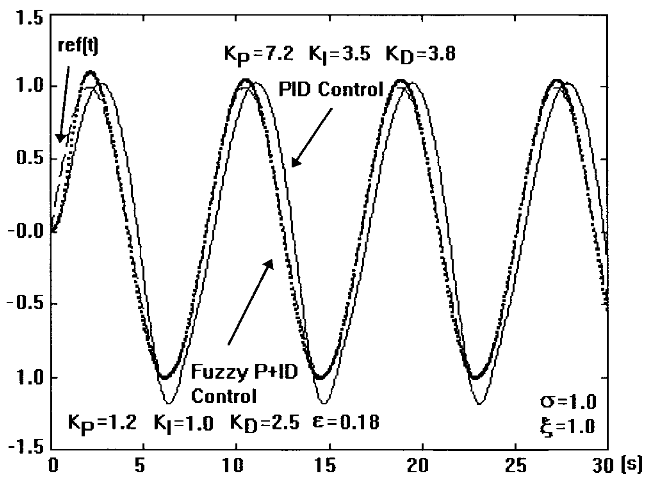
Fig. 8. Time responses of ramp-tracking control.

In the following simulations, two sinusoidal references with different frequencies are fed to the plants and the plant with $\sigma = 1.0$ and $\xi = 1.0$ are controlled by both types of the PID and fuzzy P+ID controllers. The initial values are chosen as $y(0) = 0$, $y'(0) = 0$, and $y''(0) = 0$. In Fig. 9(a) and (b), the sinusoidal signals are defined as $\mathrm{ref}(t) = \sin(0.5t)$ and $\mathrm{ref}(t) = \sin(0.575t)$, respectively. That means that the change rate of the input in Fig. 9(b) is 1.5 time of that in Fig. 9(a). It is clear that the tracking control performances becomes worse while increasing the frequencies. However, their effects on the time responses of the PID controller are much stronger than those of the fuzzy P+ID controller.

Finally, a reference signal combined by the tracking and step functions above is fed to the plant with $\sigma = 1.0$ and $\xi = 1.0$. For the PID-type controller, its parameters are chosen as ($K_{\rm P} = 7.2$, $K_{\rm I} = 3.5$, $K_{\rm D} = 3.8$) since the first part of



(a)



(b)

Fig. 9. Time responses of sinusoidal tracking control. (a) $\text{ref}(t) = \sin(0.5t)$. (b) $\text{ref}(t) = n\sin(0.075t)$.

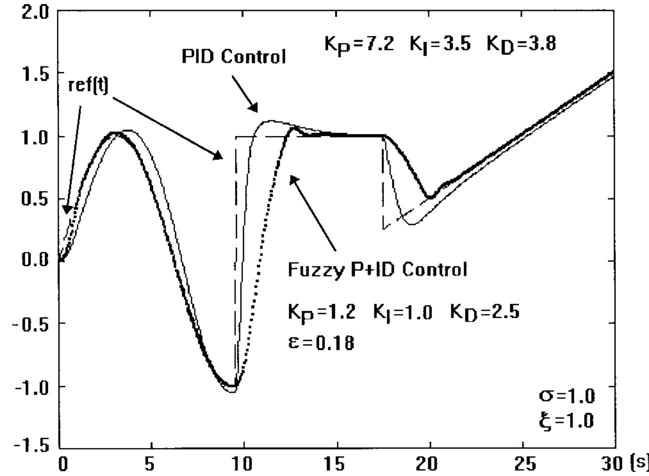


Fig. 10. Time responses of control combined by tracking and step references.

the reference is sinusoidal signal, while for the fuzzy P+ID its parameters remain unchanged. The thick-dot and thin-solid curves in Fig. 10 represent the time responses of the fuzzy P+ID and PID controllers. It can been seen that the control performance yielded by the fuzzy P+ID controller is better than that by the PID controller.

VII. Conclusions

One of the purposes for proposing the fuzzy P+ID controller is to improve control performance of many industrial plants that are already controlled by the PID-type controllers. The structure of the fuzzy P+ID controller is very simple since it is constructed by replacing the proportional term in the conventional PID controller with an incremental fuzzy logic controller. We have already applied this controller to the manipulator and stoker-fired boilers and report these application studies in our other works [6], [7].

In fact, it is very difficult to analyze the stability of many industrial plants controlled by the PID-type controller due to no any appropriate model. While implementing the fuzzy P+ID controllers for some industrial plants, such as combustion system of the stoker-fired boilers, we note that many industrial engineers do not take care of this problem very much. However, they hope that after the PID-type controller is replaced by the fuzzy P+ID controller, the systems do not change the stability situation. For this reason, we use “small gain theorem” to analyze the stability of the fuzzy P+ID controller. The study implies that if the PID controller’s parameters are defined by the sufficient stability condition in (56); the stability of the system remains unchanged after the PID controller is replaced by the fuzzy P+ID controller. In the other words, if the stability of the closed system by the PID controller can be proved, the stability of the closed system by the fuzzy P+ID controller can also be done. In fact, the condition in (55) for determination of the parameter K_{D} is very conservative, hence, K_{D} can be decreased based on control engineer experience.

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