A C-Means Clustering Based Fuzzy Modeling Method

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Abstract-**This paper proposes a new neuro-fuzzy method to model the dynamic behavior of complex systems based on real experimental data. First, we investigate the firing strength of rules by a fuzzy C-means clustering method. Then, we retrieve the membership functions of input variables by a neuro-fuzzy network. Finally, we identify the parameters of linear local models by recursive least squares. In particular, we applied this method to construct the dynamics of a boiler combustion process.**

I. INTRODUCTION

The dynamics of the most industrial plants are highly nonlinear and uncertain. Therefore, it is very difficult to investigate their dynamic behavior by using traditional modeling approaches. For several years, different fuzzy-logic-based models have been developed to cope with nonlinearity and uncertainty[1-5]. Of them, fuzzy Takagi-Sugeno-Kang (TSK) models [2-3] are widely used in control engineering. Basically, a fuzzy TSK model can be expressed by a set of following typical rules

$$\begin{aligned} \text{Rule}_l:\ & \text{IF } x_1 \text{ is } A_{l,1} \text{ AND } x_2 \text{ is } A_{l,2} \text{ AND} \cdots \text{AND } x_K \text{ is } A_{l,K} \\ & \text{THEN } y_l = a_{l0} + a_{l1}x_1 + a_{l2}x_2 + \cdots + a_{lK}x_K \\ & l = 1, 2, \cdots, L \end{aligned} \tag{1}$$

For l=1, 2, $\cdots$, L.

Where $A_{l,k}$ is one of M Gaussian membership functions associated with input variable x_k (k=1, 2,$\cdots$, K), defined by.

$$G(x) = e^{-(x-\alpha)^2/\beta^2} \tag{2}$$

A global model can be expressed by:

$$y_m = \sum_{l=1}^{L} \mu_l y_l \tag{3}$$

And

$$\mu_l = \frac{A_{l1}(x_1)\times A_{l2}(x_2)\times\cdots\times A_{lK}(x_K)}{\sum_l^L \left(A_{l1}(x_1)\times A_{l2}(x_2)\times\cdots\times A_{lK}(x_K)\right)} \tag{4}$$

μ_l in the left side of Eq.4 is called firing strength of *Rule$_l$*. In fact, Each rule defines a fuzzy cluster in the input-product space, and the corresponding firing strength associated can be regarded as the membership function of the corresponding partition. So it is possible to build a fuzzy model using a fuzzy clustering technique [6-9]. However, one of difficulties in a clustering-based fuzzy model lies in how to retrieve the appropriate membership functions from a fuzzy partition matrix related to input variables. In [6-8], a least square method is used to project the features of the membership functions onto input variables. Consequently, too many membership functions may be produced. In order to deal with this problem, an additional merge strategy is needed to reduce computation dimensions.

For building an efficient fuzzy model, it is very important to investigate the firing strength of each rule. In this paper, we propose a hybrid neuro-fuzzy approach to modeling the dynamics of complex systems in three phases. Firstly, we determine the firing strength of rules using the fuzzy c-means (FCM) clustering technique [11]. Secondly, we retrieve the membership functions related to input variables using a neural network. Finally, we identify the linear models in partitions using the recursive least squares (RLS) algorithm. In the case of a real application, we use this method to construct a dynamic model of a boiler combustion process based on the real experimental data.

II. IDENTIFICATION OF FUZZY PARTITIONS

Considering a fuzzy system with K inputs and L rules, Given a finite set of the N recorded patterns $\Omega=\{\dot{x}^n \mid \dot{x}^n=[x_1(n), x_2(n), \cdots, x_K(n)]^T \in R^K, n=1, 2,\cdots, N\}$, then firing strength about all recorded data forms a LxN matrix.

$$U = \begin{bmatrix} \mu_{11} & \mu_{12} & \cdots & \mu_{1N} \\ \mu_{21} & \mu_{22} & \cdots & \mu_{2N} \\ \vdots & & \ddots & \vdots \\ \mu_{L1} & \mu_{L2} & \cdots & \mu_{LN} \end{bmatrix} \tag{5}$$

Moreover, **U** satisfies two constrained conditions

$$\sum_{l=1}^{L} \mu_{ln} = 1,\ \mu_{ln} \in [0,1] \text{ and } 0 < \sum_{n=1}^{N} \mu_{ln} < N$$

Obviously, each row of **U** is associated with a rule and defines a fuzzy cluster in the input product space. Therefore, based on some experimental data, we can use the FCM algorithm [11] to update all fuzzy partitions in an automatic way without any initial knowledge.

III. RETRIEVE MEMBERSHIP FUNCTION

Another important task in clustering-based modeling strategy is to retrieve membership functions from obtained firing strength matrix $U=\{\mu_{ln}\}$. A commonly used method is to project the firing strength matrix onto each input variable [6-8]. As a result, for each fuzzy input variable, too many

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membership functions may be generated, so that the computation complexity is increased. Besides, the implementation is difficult, and the generalization ability is poor.

Actually, for an input pattern, its membership degree in a cluster is just the firing strength of the corresponding fuzzy rule defined by Eq.4. Hence, it is possible to retrieve the membership functions of input variables by setting the functions of Eq.4 equal to μ_{ln}.

If we consider all possible combinations, Eq.4 can be easily represented as a seven-layered Gaussian neuro-fuzzy network with K inputs and $L=M^K$ outputs, as shown in Fig.1.

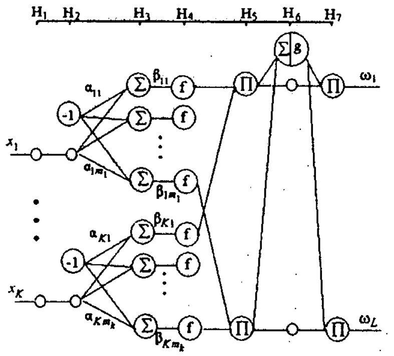

Fig. 1: Neural network for identifying membership functions of input variables

Therefore we can determine the α, β parameters of the membership functions through a learning process. Here, we use their corresponding firing strength μ_{ln} as the reference outputs of the network and train the retrieval network by minimizing the following error function

$$E=\frac{1}{2}\sum_{n=1}^{N}\sum_{l=1}^{L}(\mu_{ln}-\omega_{ln}^{model})^2 \tag{7}$$

where ω_{ln}^{model} is the real output of the network when $\hat{x}^n$ is fed to the network. According to the δ learning rule, the BP algorithm for training the network can be deduced below:

$$\begin{cases} w_{i,j}^{h-1,h}(t+1)=w_{i,j}^{h-1,h}(t)+\eta\delta_j^h o_i^{h-1} \\ \qquad +\gamma\,\Delta w_{i,j}^{h-1,h}(t) \qquad \textit{for Case A} \\ w_{i,j}^{h-1,h}(t+1)=w_{i,j}^{h-1,h}(t) \\ \qquad +\eta\delta_j^h o_i^{h-1}*\left(\prod_{s\neq i} w_{sj}^{h,h+1}o_s^h\right) \\ \qquad +\gamma\,\Delta w_{i,j}^{h-1,h}(t) \qquad \textit{for Case B} \end{cases} \tag{8}$$

for h=1, 2, ⋯, 7

where, $w_{i,j}^{h-1,h}$ is the connection weight from the ith neuron in the $(h-1)$th layer to jth neuron in the hth layer, o_i^h is the output of the ith neuron in the layer h. The parameter δ is computed as follows:

For the neurons in the output layer:

$$\delta_j^7=f'(net_j^7)E \tag{9}$$

For the neurons in hidden layers:

$$\begin{cases} \delta_j^h=f'(net_j^h)\sum_s \delta_j^{h+1}w_{js}^{h,h+1} \\ \qquad \textit{for Case A} \\ \delta_j^h=f'(net_j^h)\sum_s \delta_j^{h+1}w_{js}^{h,h+1}*\left(\prod_{i\neq j} w_{is}^{h,h+1}o_i^h\right) \\ \qquad \textit{for Case B} \end{cases} \tag{10}$$

where net_j^h is the input to the jth neuron in the layer h. In Eqs.8 and 10, *Case A* denotes all neurons aggregated by the summation operation, and *Case B* denotes all neurons aggregated by the product operation.

IV IDENTIFICATION OF LOCAL MODEL PARAMETERS

After obtained the firing strength of the rules, we can rewrite the fuzzy model defined by Eq.3 as:

$$z^n=\sum_{l=1}^{L}\mu_l^n y_l^n+e^n=\hat{\theta}^T\hat{\psi}^n+e^n \tag{11}$$

where $\hat{\theta}$ and $\hat{\psi}^n$ are the local model parameter vector and a vector with the dimension of dim= $L\times(K+1)$ defined by

$$\hat{\theta}=(a_{10},a_{11},\cdots,a_{1K},a_{20},a_{21},\cdots,a_{2K},\cdots, a_{L0},a_{L1},\cdots,a_{LK})^T \tag{12}$$

$$\hat{\psi}(n)=(\mu_{1n},\mu_{1n}x_1(n),\cdots,\mu_{1n}x_K(n), \mu_{2n},\mu_{2n}x_1(n),\cdots,\mu_{2n}x_K(n),\cdots, \mu_{Ln},\mu_{Ln}x_1(n),\cdots,\mu_{Ln}x_K(n))^T \tag{13}$$

Given a set of N recorded data Ω={ $\hat{x}^n$ |n=1, 2,⋯, N}, we can identify the model parameters by using the RLS algorithm [13].

V DYNAMIC BEHAVIOR MODELING OF STOKER-FIRED BOILER

A stoker-fired boiler, as shown in Fig. 2, is an important and widely used means of power generation. Due to the lack of an accurate model of the boiler dynamics, we only can design controllers based on control engineer experience [8]. In particular, due to a large time constant, high uncertainty, nonlinearity of the system, large disturbances and so forth, tuning of the parameters of such controller is time consuming work during boiler operation. If we could build a model of the boiler dynamics, which describes the combustion process of the boiler accurately, we can investigate its dynamic behavior off-line [12]. Our experience shows that this can greatly reduce the time for controller design and tuning.

Based on our previous work [4][12], the dynamics of the boiler can be approximately described by the nonlinear difference equation:

$$\begin{aligned} pr(n) = f(&pr(n-1),\ pr(n-2), co(n),\ co(n-1), \\ &ar(n), ar(n-1), fw(n), fw(n-1)) \end{aligned} \tag{14}$$

where, *pr*, *cl*, *ar* and *fw* stand for steam pressure, coal feed, air feed and steam flow, respectively. In order to model the dynamics of the boiler, we approximate Eq.14 using a fuzzy model. The fuzzy model proposed for the boiler dynamics has eight fuzzy input variables. They are pressure, air, coal and flux at the time instants indicated in Eq.14 Each fuzzy variable is assigned with three fuzzy subsets labeled "*Small*", "*Medium*" and "*Large*", respectively.

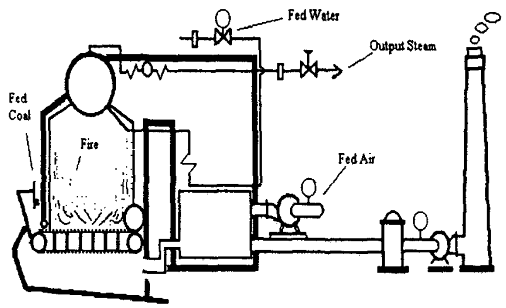

Fig.2: A stoker-fired boiler

Usually, the number of rules in the rule base increases exponentially with the number of fuzzy variables in a fuzzy system. If we consider all combinations of fuzzy variables, 3^8=6,561 rules will be produced in the rule base. A large number of rules can lead to a retrieval network with large size, difficulty of implementation, and complexity of computation. Many rules in rule base are either similar to each other, or have small influence on the output. Deleting such redundant rules should not spoil the performance of model. Simplification of rule base, however, often involves our knowledge of the dynamics of a real plant to be modeled. In this paper, we utilize two known facts:

1. An air feed amount is usually proportional to a coal feed amount to make coal completely burned, hence we delete rules with inconsistent fuzzy sets between the coal feed and the air feed.

2. In the continuous running of a boiler, a rule should not be constructed just by one fuzzy variable from a same physical quantity. For example, steam pressure *pr*(*n*-1) or *pr*(*n*-2) at instant *n* or *n*-1 will not be combined. In this case, we can delete the connections between each couple of variables with the same physical quantity in the rule base.

By such a simplification, there only exist 54 rules in the rule base, and they can be equally classified into two categories:

Group one:

$$\textit{If } cl(n)\,(ar(n)) \textit{ is } \begin{Bmatrix} \textit{Large} \\ \textit{Medium} \\ \textit{Small} \end{Bmatrix} \textit{ and } fw(n) \textit{ is } \begin{Bmatrix} \textit{Large} \\ \textit{Medium} \\ \textit{Small} \end{Bmatrix}$$

$$\textit{and } pr(n-1) \textit{ is } \begin{Bmatrix} \textit{Large} \\ \textit{Medium} \\ \textit{Small} \end{Bmatrix}$$

$$\begin{aligned} \textit{Then } pr(n) = a_{i0} &+ a_{i1}cl(n) + a_{i2}cl(n-1) \\ &+ a_{i3}ar(n) + a_{i4}ar(n-1) \\ &+ a_{i5}fw(n) + a_{i6}fw(n-1) \\ &+ a_{i7}pr(n-1) + a_{i8}pr(n-2) \end{aligned}$$

Group two

$$\textit{If } co(n-1)(ar(n-1)) \textit{ is} \begin{Bmatrix} \textit{Large} \\ \textit{Medium} \\ \textit{Small} \end{Bmatrix} \textit{ and } fw(n-1) \textit{ is} \begin{Bmatrix} \textit{Large} \\ \textit{Medium} \\ \textit{Small} \end{Bmatrix}$$

$$\textit{and } pr(n-2) \textit{ is } \begin{Bmatrix} \textit{Large} \\ \textit{Medium} \\ \textit{Small} \end{Bmatrix}$$

$$\begin{aligned} \textit{Then } p(n) = a'_{i0} &+ a'_{i1}co(n) + a'_{i2}co(n-1) \\ &+ a'_{i3}ar(n) + a'_{i4}ar(n-1) \\ &+ a'_{i5}fw(n) + a'_{i6}fw(n-1) \\ &+ a'_{i7}pr(n-1) + a'_{i8}pr(n-2) \end{aligned}$$

Now, we build the fuzzy dynamic model of a boiler in a Chemical Plant by using its 700 operation data. A sampling time for recording operation data of the boiler system is 1 minute. It should be noted that the premise of each rule defines a partition of the input product space. Considering the boiler model deduced above, each rule in Group one is concerned with pressure at instant *n*-1, air, coal and steam flux amount in the instant *n*, and it defines fuzzy partition in the Cartesian product space (**PS**)

$$\mathrm{PS}_1 = pr(n-1) \times ar(n) \times co(n) \times fw(n) \subset \mathrm{R}^4$$

The corresponding firing strength of rules in Group one can be obtained by FCM in the space $\mathbf{PS}_1$. Similarly, each rule in Group two is related to pressure at instant *n*-2, air, coal and steam flux at the instant *n*-1, and it also defines another partition of the product space

$$\mathrm{PS}_2 = pr(n-2) \times ar(n-1) \times co(n-1) \times fw(n-1) \subset \mathrm{R}^4$$

The corresponding firing strength of a rule in Group two can be obtained by FCM in the product subspace $\mathbf{PS}_2$. Based on the obtained firing strength matrix of 54×700. The outputs of the fuzzy model and of the boiler are shown in Fig. 3. In order to evaluate the fuzzy dynamic model, we use the mean squared error (MSE) as a performance index

$$\mathrm{MSE} = \overline{E}^2 = \frac{1}{N}\sum_{n=1}^{N}(z^n - out^n)^2 \tag{15}$$

where z^n (n=1, 2, ⋯, N) is a reference output of fuzzy model, and out^n denotes its real output.

In order to demonstrate the effectiveness of the proposed model, Fig. 4 plots $PI(n)$ vs n, where $PI(n)=\left(z^n-out^n\right)^2$. Note that the measured pressure, not estimated pressure, is used as an input to the fuzzy system in the evaluation. It can be seen from Fig. 3 and Fig. 4 that the obtained fuzzy model describes the dynamic behavior of combustion process of the boiler with good performance.

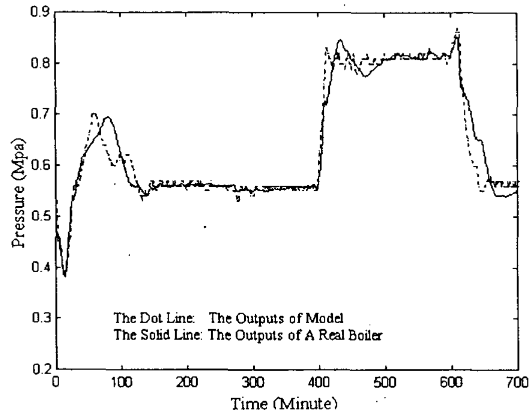

Fig. 3: Fuzzy model of boiler

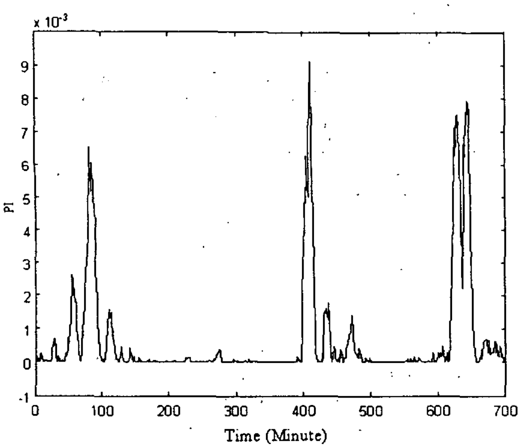

Fig. 4: Error between the measured dynamics and the one yielded by the fuzzy model.

VI. CONCLUSIONS

The fuzzy model is widely used in the control community, but how to build an efficient fuzzy model is a difficult problem. In this paper, we proposed a new fuzzy model identification method, which combines the clustering-based fuzzy modeling method with neural network techniques. Compared with conventional clustering-based identification methods, our method can easily retrieve the membership functions for input variables and can generate a small number of fuzzy subsets for each fuzzy variable. In the presented work, we also model a real boiler using the proposed method, and the result demonstrates the effectiveness of this new method. In our application to boiler modeling, we also discuss the method how to rationalize the rule base according to knowledge about boiler operations.

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